Maximizing RF Signal in the Presence of Rapid T2 Relaxation

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Introduction: Traditional MR imaging is predominantly geared towards long T_2 species, for which the RF duration τ can be considered negligible compared to the intrinsic T_2. When using UTE methods to image short or ultra short T_2 species, such as ligaments, tendons or cortical bone, the intrinsic T_2 can be on the same order as τ, and the signal decay during the RF pulse may no longer be ignored [1,2]. In this work we show how to select a nominal flip angle (for a given maximum B_1) that maximizes signal amplitude for these circumstances. In addition, we derive an analytic expression for an effective TE for short T_2 species.

Single RF Pulse: The two parameters under the control of the pulse programmer that determine the flip angle are the magnetic RF field strength B_1 and pulse duration τ, which for a hard RF pulse leads to $\theta = \gamma B_1 \tau$ for the nominal flip angle. Solving the Bloch equations for the transverse and longitudinal magnetization from a single hard RF pulse along the x-axis of magnetic field strength B_1 ($\omega_1 = \gamma B_1$) in the presence of T_2 relaxation leads to [2]:

$$M_y(\tau) = M_0 \frac{\omega_1}{\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}} \exp\left(-\frac{\tau}{2T_2}\right) \sin\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right) \quad \text{and} \quad M_z(\tau) = M_0 \exp\left(-\frac{\tau}{2T_2}\right)\left[\cos\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right) + \frac{1}{2T_2\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}} \sin\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right)\right] \tag{1}$$

A plot of $M_z(\tau)$ vs. $M_y(\tau)$ for the case of constant B_1 = 25μT and θ = 180° is shown in Fig.1. Note the evolution of the magnetization corresponds to an inversion only for the limiting case of $T_2 \rightarrow \infty$. As noted in [1], the maximum of the signal M_y generally does not occur at θ = 90° (marked on the curves as solid dots), but at a smaller flip angle (e.g. dashed line). Setting the derivative of M_y with respect to τ to zero yields an expression for the nominal flip angle that maximizes signal, given in Eq.2. Fig.2 shows Eq.2 as a function of T_2 for different values of B_1. As expected, the value approaches 90° for long T_2 species but is less than 90° for shorter T_2 species.

Optimum flip angle for single RF pulse:

$$\theta = \gamma B_1 \left[\frac{\arctan\left(2T_2\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\right)}{\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}}\right] \tag{2}$$

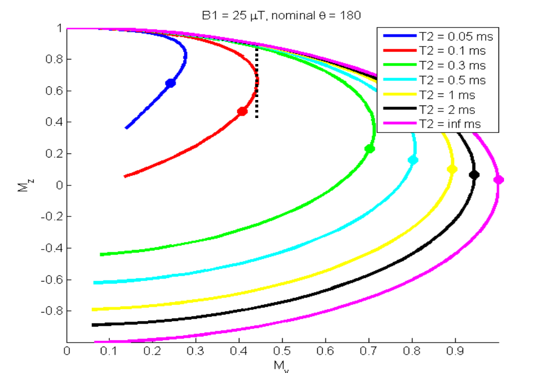

Fig.1: Longitudinal vs. transverse magnetization for a single hard RF pulse with a fixed B_1 = 25 μT, and a nominal 180° flip angle.

Effective Echo Time (TE): Ordinarily, the echo time *TE* is defined as the time from the center of the RF pulse to the time of the echo formation of the MR signal [3]. If we denote the time from the end of the RF pulse to the time of echo formation as the wait time T_w, then $TE = T_w + \tau/2$. Robson et al. studied the effects of signal decay when T_2 is on the order of τ [4]. They concluded that the echo time is generally a function of T_2, i.e. $TE(T_2)$, with asymptotic behaviors: $TE(\infty) = T_w + \tau/2$ and $TE(0) = T_w$. The notion of an effective $TE(T_2)$ may be solidified by casting M_y of Eq.1 into the form of a traditional exponential T_2 decay, which leads to Eq.3. This translates T_2 decay occurring during the RF pulse into the more familiar concept of echo time. A plot of the effective *TE* as a function of T_2 for different values of θ is shown in Fig.3.

Effective echo time:

$$TE(T_2) = T_w + \frac{\tau}{2} - T_2 \ln\left[\frac{\mathrm{sinc}\left(\sqrt{\theta^2 - \frac{\tau^2}{4T_2^2}}\right)}{\mathrm{sinc}(\theta)}\right] \tag{3}$$

Spoiled RF Pulse Train: The previous analysis considered the case of a single RF pulse with fully relaxed spins ($M_z = M_0$), which is equivalent to $TR >> T_1$. In practice, the longitudinal magnetization is not usually allowed to re-grow to its equilibrium value M_0, so we must also consider the case of spoiled RF pulse train acquisition. Combining the two equations of Eq.1, an expression for the steady-state magnetization M_{ss}, is readily obtained:

Steady State Magnetization:

$$M_{SS} = M_0 \frac{\omega_1\left(1-\exp\left(-\frac{TR}{T_1}\right)\right)\exp\left(-\frac{\tau}{2T_2}\right)\sin\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right)}{\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\left\{1-\exp\left(-\frac{TR}{T_1}\right)\exp\left(-\frac{\tau}{2T_2}\right)\left[\cos\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right) + \frac{1}{2T_2\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}}\sin\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right)\right]\right\}} \tag{4}$$

Taking the derivative of Eq.4 with respect to τ and equating to zero yields the optimal nominal flip angle ($\theta = \gamma B_1 \tau$) for a given B_1:

Generalized Ernst Angle for SPGR:

$$\cos\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right) - \frac{1}{2T_2\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}}\sin\left(\sqrt{\omega_1^2 - \frac{1}{4T_2^2}}\,\tau\right) = \exp\left(-\frac{TR}{T_1}\right)\exp\left(-\frac{\tau}{2T_2}\right) \tag{5}$$

Note that Eq.5 reduces to the classical Ernst angle in the limit $T_2 \rightarrow \infty$. Fig.4 shows the optimum flip angle vs. T_2 for several values of TR/T_1 and fixed B_1 = 25 μT.

Conclusion: We have investigated the use of single hard RF pulses and pulse trains for imaging short T_2 species. We have derived an analytical expression for the effective *TE* that is consistent with the earlier work, and an expression that determines the optimum nominal flip angle for maximizing signal from different T_2 species.

References: [1] Tyler et al, JMRI 25:279 (2007) [2] Sussman et al, MRM 40:890 (1998) [3] ACR Glossary of MR Terms (1995) [4] Robson et al, ISMRM 11 (2004)

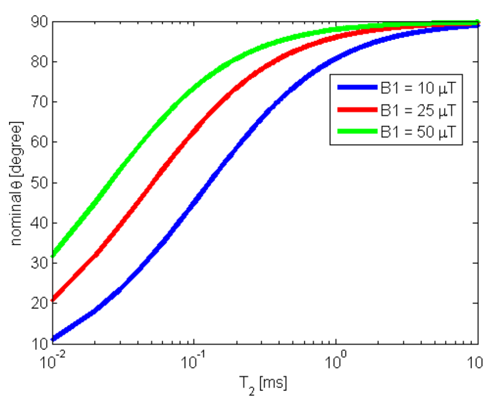

Fig.2: Optimum nominal flip of a single hard RF pulse as a function of T_2.

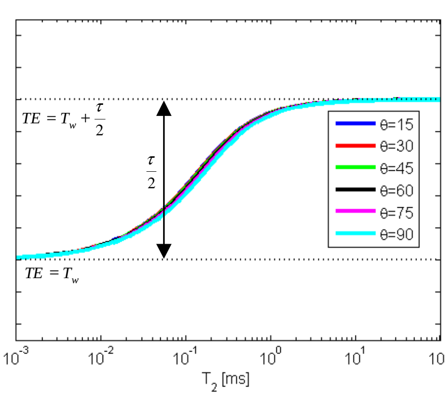

Fig.3: TE vs. T_2 for different flip angles using a τ = 1ms RF hard pulse.

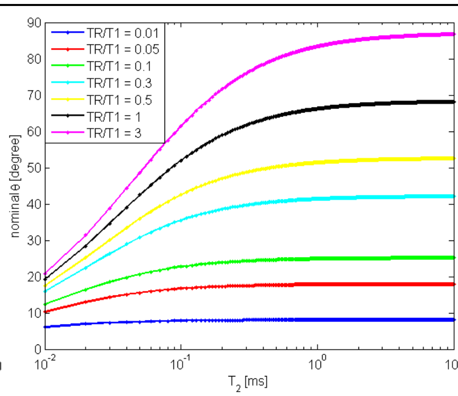

Fig.4: Optimum nominal flip angle of a hard RF pulse train as a function of T_2.