

# Induced Correlation In fMRI Magnitude Data From k-Space Preprocessing

D. B. Rowe<sup>1</sup>, and A. S. Nencka<sup>1</sup>

<sup>1</sup>Department of Biophysics, Medical College of Wisconsin, Milwaukee, WI, United States

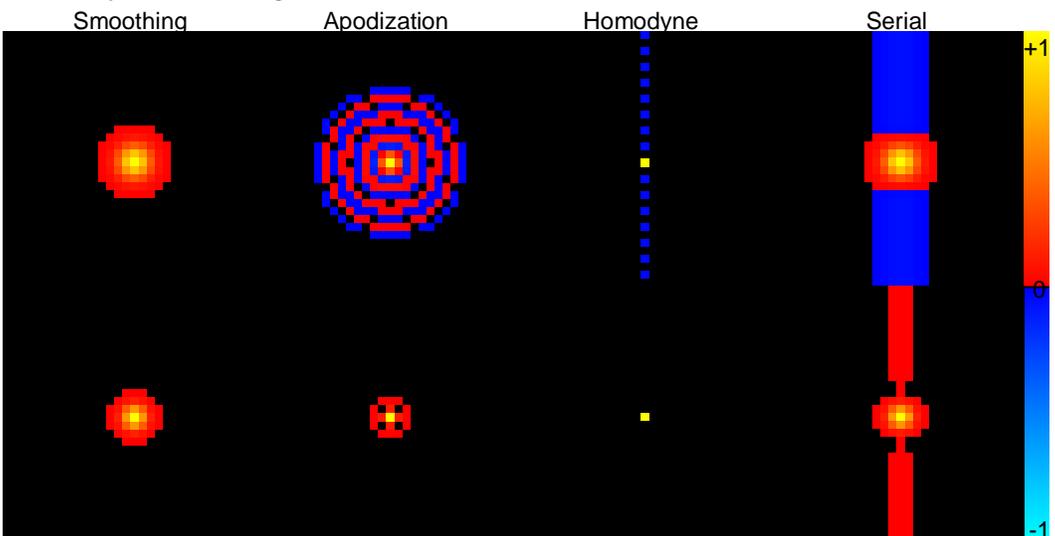
**Introduction:** Correlations between image-space voxels over time have been used to identify functionally connected regions of the cortex of subjects in the resting state (1). Such analysis assumes that image-space voxel correlations arise only from physiologic fluctuations. Much work has been done to temporally filter the image-space voxel time series to frequency windows in which voxel correlations arise from functional physiologic correlations. However, little consideration has been made of the commonly used image processing techniques which necessarily induce voxel correlations in image-space. Such image processing techniques, however, alter the observed time series correlations as one voxel's signal may be spread over several voxels. Some work has been done to consider the contributions of common image processing techniques on the correlations of the complex-valued image-space observations (2,3). This extends the previous work to the more relevant correlations within the magnitude-squared data, which are asymptotically equivalent to the magnitude correlations, as most correlation studies consider magnitude data.

**Theory:** It has been shown that correlations in complex-valued image-space data caused by common image preprocessing can be determined by linear algebra (2). Consider the complex-valued image data with a real-valued isomorphism of a vector of real observations stacked above imaginary observations (4). Let  $O_k$  be a linear operation performed on the complex-valued k-space observations,  $\Omega$  be a Fourier reconstruction matrix, and  $O_r$  be a linear operation performed on the reconstructed complex-valued image-space observations. Then if  $s_0$  and  $\Gamma$  are the true mean and covariance matrix of the k-space observations, the resulting image-space mean image is  $\mu = O_r \Omega O_k s_0$  and covariance matrix is  $\Sigma = O_r \Omega O_k \Gamma O_k^T \Omega^T O_r^T$ . This image space covariance matrix can be written as  $\Sigma = [\Sigma_R, \Sigma_{RI}; \Sigma_{RI}^*, \Sigma_I]$  where  $\Sigma_R$  is the within real observations covariance matrix,  $\Sigma_I$  is the within imaginary observations covariance matrix, and  $\Sigma_{RI}$  is the between real and imaginary observations covariance matrix. Define  $A_{jj} = [\Sigma_{Rjj}, \Sigma_{RIj}; \Sigma_{RIj}^*, \Sigma_{Ijj}]$  and  $B_{jk} = [\Sigma_{Rjk}, \Sigma_{RIjk}; \Sigma_{RIjk}^*, \Sigma_{Ijk}]$ . Then assuming normally distributed k-space observations, the mean of a magnitude square observation in voxel  $j$  is  $\tau_j = tr(A_{jj}) + \mu_j' \mu_j$ , the variance is  $\Lambda_{jj} = 2tr(A_{jj} A_{jj}') + 4\mu_j' A_{jj} \mu_j$ , and the covariance between voxels  $j$  and  $k$  is  $\Lambda_{jk} = 2tr(B_{jk} B_{jk}') + 4\mu_j' B_{jk} \mu_k$ . In the above,  $\mu_j = (\rho_j \cos \theta_j, \rho_j \sin \theta_j)'$  is the reconstructed real and imaginary observations of voxel  $j$  with magnitude and phase  $\rho_j$  and  $\theta_j$ .

To examine the effects only from image processing, the image data is examined with a k-space mean of zero ( $s_0 = 0$ ) and identity covariance matrix ( $\Gamma = I$ ). Thus the image-space covariance matrix simplifies to  $\Sigma = P \Omega O O^T \Omega^T P^T$ . For clarity of presentation, the case where  $\Sigma_{Rjj} = \Sigma_{Ijj}$  is considered. The correlation matrices are computed as  $R_\Sigma = D_\Sigma^{-1/2} \Sigma D_\Sigma^{-1/2}$  and  $R_\Lambda = D_\Lambda^{-1/2} \Lambda D_\Lambda^{-1/2}$  where  $D_\Sigma$  and  $D_\Lambda$  are diagonal with variances from  $\Sigma$  and  $\Lambda$ .

**Methods:** Linear operators for three common k-space image processing techniques were created. A 32x32 image acquisition matrix was considered. These operators include: a Gaussian smoothing filter with an image-space FWHM of 1.1774 voxels; a Tukey apodization filter commonly used in spiral reconstruction with a plateau width of 12 voxels and a slope width of 4 voxels; and an operator to perform extrapolation of the symmetric half of k-space as is common in partial k-space acquisitions. The complex data image-space covariance matrix  $\Sigma$  and corresponding magnitude squared covariance matrix  $\Lambda$  were computed and correlation matrices  $R_\Sigma$  and  $R_\Lambda$  were determined from them. Image-space correlations for the center voxel after applying the processes individually and serially are shown in Figure 1.

**Results:** Correlations from k-space pre-processing are as expected. From the above equations, magnitude squared correlations are less than complex correlations when  $\Sigma_{RIjk} < 0.125$ , and the complex data variance is one. When simple convolution is applied as in the cases of smoothing and apodization, the image-space correlations reflect convolution with the Fourier transform of the k-space kernel. This method allows the examination of induced image-space correlations from nonintuitive processes. It is seen that slight correlations in the phase encode direction are induced by homodyne reconstruction.



**Figure 1** Image-space correlations for center voxel in real and magnitude squared data. Center 2 of 256 colorbar values black.

**Conclusion:** The image-space correlations that are induced by multiple image-processing steps can be easily considered in this framework. This theoretical work provides the basis for future work to improve functional connectivity studies. By quantifying the image-space correlations caused by pre-processing methods, the pre-processing induced correlations can be removed and separated from the true biological correlations. After removal of induced correlations, cleaner biological correlations will remain. This will enhance and refine all future fMRI connectivity studies.

**References:** 1. Biswal, Yetkin, Haughton, Hyde MRM 34:537-541, 1995. 2. Nencka, Rowe. ProcOHBM S55:284, 2007. 3. Nencka, Rowe ProcISMRM 2457, 2008. 4. Rowe, Nencka, Hoffmann JNeurosciMeth. 159:361-369, 2007. **Support:** Funded in part by NIH EB00215, MH019992.