Deconvolving Haemodynamic Response Function in fMRI under high noise by Compressive Sampling

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Introduction. A simple technique to deconvolve haemodynamic response function (HRF) from fMRI data using 1-norm minimization is introduced. The true HRF is typically sparse after wavelet transform, but we find the proposed technique to be robust w.r.t relative sparsity. HRF is deconvolved via convex optimization which has the flexibility to impose local HRF monotonicity and smoothness in the time domain. Real fMRI data under low SNR (-10dB) confirms reliability of this technique.

Methods. Canonical HRF is often modeled using gamma functions (1)-(3), but subject-specific HRF may be desirable in studies of subjects like children and stroke victims who have noncanonical HRF (4)(5). HRF can be measured by applying a short stimulus to invoke it; excitation is repeated at suitably long intervals for better statistics. But this approach is time consuming and inefficient, not every stimulus can be implemented impulsively (*e.g.* emotion, memory), and there are tenuous assumptions about cross-modal stimuli and HRF homogeneity across brain regions. The technique proposed herein instead extracts HRF at once from all the actual data comprising multiple design-stimulus onsets. An impulsive stimulus is used only to validate the proposed technique. The true HRF is assumed sparse under wavelet transform. Coiflet 4 wavelet is chosen because, empirically, it gives the sparsest transform of canonical HRF. The proposed technique finds HRF by solving the following optimization problem:

$$\begin{aligned}\underset{h}{\text{minimize}}\quad & \|Wh\|_1 + \lambda_1 \|y - Dh\|_2 + \lambda_2 \|\nabla^3 h\|_1 \\ s.t.\quad & h(1) = h(n) = 0 \\ & E^{\mathrm{T}} h \le 0\end{aligned}$$

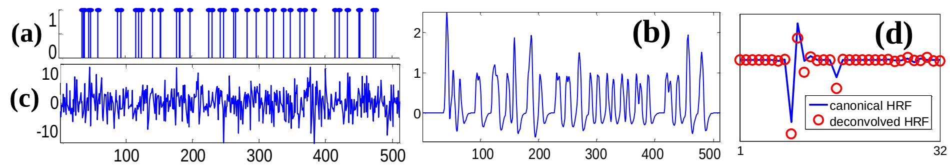

Figure 1. Simulation. **(a)** ER design with jittered impulses. **(b)** Convolution of ER design and canonical HRF. **(c)** Measurement data = Gaussian random noise + (b). **(d)** Wavelet coefficients of the canonical and a deconvolved HRF.

where h is the unknown HRF of length n, Wh is the discrete wavelet transform of h, vector y is the fMRI data, D is a convolution matrix holding lagged design stimuli, and ∇^3 is a third-order gradient. To admit noise and to impose a smoothness constraint, additional constraints appear in the objective governed by λ_1 and λ_2; empirically determined positive scalars. The tail of HRF can be constrained nondecreasing by requiring that it belong to the *monotone nonnegative convex cone* represented by matrix E. (6,ch.2)

In simulation, a block design stimulus consists of nine on/off blocks each with period 60 in a length-512 record. A separate event-related (ER) design consists of 40 uniformly or randomly-placed (jittered) impulses along a length-512 record. All design stimuli (value 0 or 1) are convolved with canonical HRF (2). Gaussian random noise is added such that SNR = +6dB and -10dB. Figure 1 shows an example of ER jittered-design, with high noise (SNR = -10dB), and the wavelet coefficients of the canonical and a deconvolved HRF. Each simulation was run 200 times; median value and standard deviation are plotted in Figure 2. Error, defined as $\varepsilon \equiv 20\log_{10}\left(\|h_{\text{in}} - h\| / \|h_{\text{in}}\|\right)$ between the input canonical HRF (h_{in}) and deconvolved HRF (h), is also listed in Figure 2.

Physical experiments were carried out on a 1.5T (GE, Signa). Two experiments were performed for HRF deconvolution: First, a block-design experiment (30s on/30s off, 9 cycles) using a checkerboard stimulus. Second, an ER-jittered motor-experiment (40 jittered tones each queuing subject to press button once with index finger). Before each experiment, HRF was directly measured by applying the appropriate corresponding impulse-stimulus once per 30s interval. This measurement cycle was repeated 20 times, then synchronously averaged over all stimulus-cycles to generate a *reference HRF*. Scanning parameters were: TE=40ms, TR=1s, slice thickness=4mm, gap=1mm, #slices=8, image-matrix size=64x64.

Results. Two regions of interest (ROI, one visual, one motor) are chosen to demonstrate results from real data. Figure 3 shows the pixelwise median reference-HRF and its standard deviation (gray bounds) for each ROI. Time-series from the block- and ER-design, for the corresponding ROI, were extracted to deconvolve HRF by the proposed convex optimization technique. The median-deconvolved HRFs and standard deviation bars are overlaid on the respective reference-HRF in Figure 3. Apparently, deconvolved HRF using our technique agrees well with the reference HRF. This shows that our technique can estimate HRF from actual scan-data just as well as it can by performing an impulsive HRF measurement; in other words, we can dispense with the impulsive measurement phase in fMRI.

Discussion. This simple technique reliably deconvolves underlying HRF in block and ER experiments. The reference measurement of HRF in a prescan commonly performed in fMRI, therefore, can be eliminated. The proposed technique is independent of the nature of noise corrupting the HRF. What is remarkable is the magnitude of noise **(**well in excess of 0dB**)** tolerated by the algorithm in its recovery of HRF. Application of convex optimization to this deconvolution problem allows the introduction of constraints on local monotonicity, endpoints, and smoothness while simultaneously satisfying a sparsity objective in the wavelet domain.

References. 1. Friston et al., NeuroImage 1998, 7(1) P30-40. 2 Glover, NeuroImage 1999, 9(4) P416-429. 3. Worsley et al., 2002, 15(1) P1-15. 4. Thomason et al., NeuroImage 2005, 25(3) P824-837. 5. Pineiro et al., Stroke 2002, 33 P103-109. 6. Dattorro, Convex Optimization & Geometry, Meboo, 2005.
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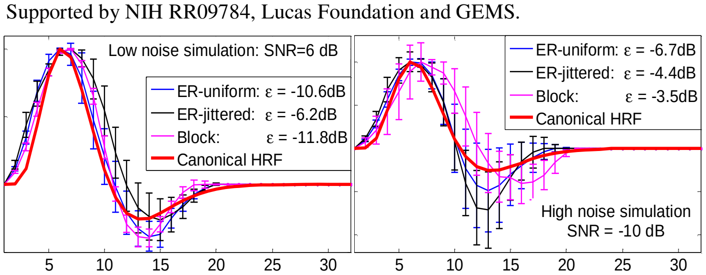

Figure 2. Simulation results under low and high noise compared with ideal canonical HRF. Three types of inputs are used: ER with uniform impulses, ER with jittered impulses, and block design. Median results and standard deviation from 200 runs are plotted.

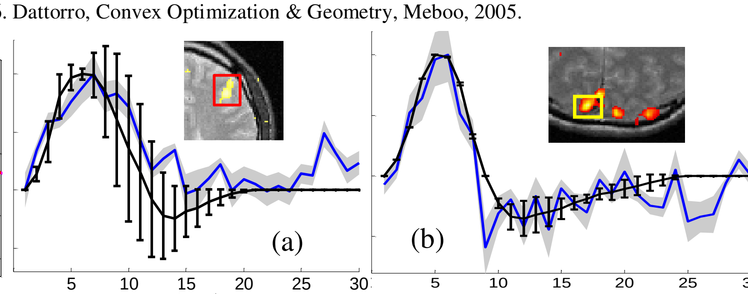

Figure 3. Experimental results: Pixelwise-median reference-HRF (blue) and standard deviation (gray bounds). Pixelwise-median deconvolved HRFs (black line and standard deviation bars) from **(a)** ER-jittered stimulus and **(b)** block visual stimulus.