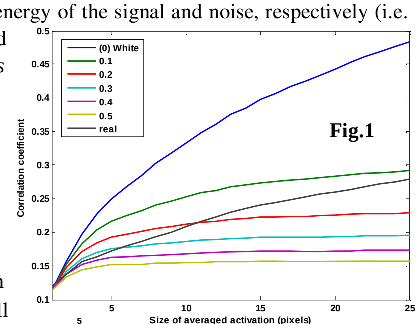
Effectiveness of Gaussian Smoothing on Spatially Correlated Noise: A 3T Case Analysis

M. Jin[1], R. Nandy[2], and D. Cordes[1]

[1]Radiology, University of Colorado Denver, Aurora, Colorado, United States, [2]Biostatistics and Psychology, UCLA, Los Angeles, CA, United States

Introduction Gaussian spatial smoothing using a fixed FWHM is a common preprocessing step in fMRI data analysis [1]. The usefulness of this step relies on the assumption that the activation patterns are convex and larger than the width of the smoothing kernel and that the noise features have a sufficient randomness. When both of these assumptions are satisfied, the SNR is being increased with smoothing and the number of activations detected will be larger than without smoothing. However, none of these assumptions are sufficiently satisfied for common fMRI data since shapes of activations are confined to gray matter which due to its folding has a complicated geometry and also there is significant intrinsic spatial correlation of the noise. Thus, the usefulness of spatial smoothing is rather questionable. In this work, we investigate spatial correlations in the noise of typical 3T fMRI data and show that common spatial smoothing can adversely degrade the detection rate of activation.

Theory Suppose a deterministic variable s is corrupted by a random (noise) variable n having zero mean, and there are p observations of s. Denote each observation by $s_i=s+n_i$ for $i=1\ldots p$. Then the estimator $s^*=\Sigma_i s_i/p$ will give $p^{1/2}$ times the SNR of that belonging to a single observation s_i when the n_i are uncorrelated. Now, let the signal s be a random variable and let the noise and signal be uncorrelated with each other, and $S^2=E[s^2]$ and $N^2=E[n^2]$ be the energy of the signal and noise, respectively (i.e. SNR=S/N). Furthermore, let $s_i=s+n_i$ be the measured signal at spatial position i, and let the noise be spatially correlated by $E(n_i n_j)=c^2\neq 0$ and $c^2<N^2$ where $i\neq j$. Then the correlation of s and s_i is $\langle s|s_i\rangle=1/(1+N^2/S^2)^{1/2}$ whereas the correlation of s and the average of p pixels gives the expression $\langle s|1/p\ \Sigma_i s_i\rangle=1/(1+1/p\ N^2/S^2+(p-1)/p\ c^2/S^2)^{1/2}$. Clearly, $\langle s|s_i\rangle$ is smaller than $\langle s|1/p\ \Sigma_i s_i\rangle$ because of the signal averaging resulting in decreased variance in the denominator (compare term $1/p\ N^2/S^2$ in $\langle s|1/p\ \Sigma_i s_i\rangle$ with term N^2/S^2 in $\langle s|s_i\rangle$). In addition, with increased correlation in the noise, the variance in the denominator of $\langle s|1/p\ \Sigma_i s_i\rangle$ increases and the detection of the averaged signal is less efficient but still more efficient than without averaging. However, taking edge effects into consideration (since the spatial filter with fixed size is not guaranteed to cover only the activation), the combination of edge effects and spatial correlations in the noise may lead to less signal detection.



Methods and results In order to quantify the influence of spatial correlations in the noise on the detection of activation by spatial smoothing, we generate spatially correlated artificial noise to satisfy $E[y_p y_q]=c^2$ ranging from $c^2=[0, 0.5]$ for all $p\neq q$ (invariant with distance and angle) and $E[y_p y_q]=1$ for $p=q$. Then, synthetic activations A with different shape and size (1 to 25 pixels) were added onto artificial noise or real resting state data Y as $X=fY+(1-f)A$, where f(=0.9 in this study) is a factor describing the noise content in the data Y. The temporal profile of activated pixels is modeled by a boxcar function (22 sec on, 22 sec off, 10 periods) convolved with the canonical (bi-gamma) hemodynamic response function.

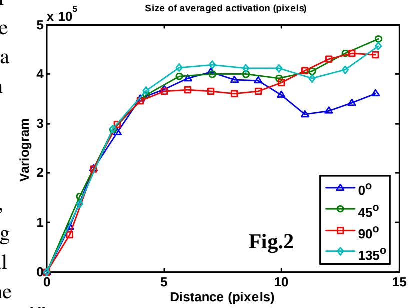
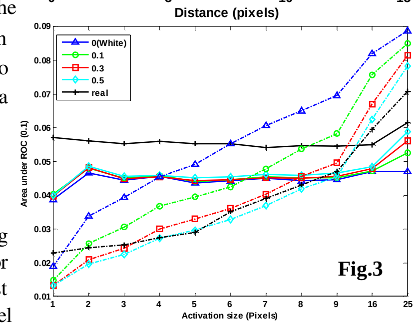
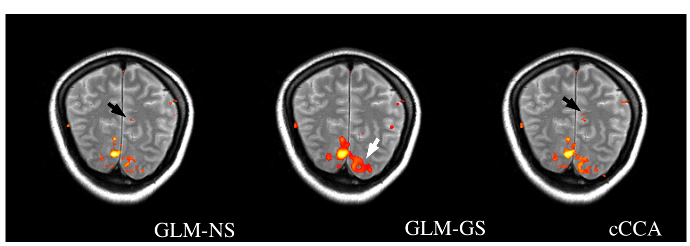


Fig. 1 shows the detection power measured by the correlation coefficient of the activated region with the assumed temporal activation (reference) function for 6 artificial data sets compared with that using resampled resting-state data as noise (GE 3T scanner, FOV 22, 128×128 resolution, thickness/gap 4mm/1mm, 25 slices, 220 time frames, FA 70, TE/TR 30ms/2s). Each data point in these curves was obtained by averaging 100 realizations. It is apparent that averaging (smoothing) over the activation pixels is very effective in improving the detection power of noise-contaminated signal when the noise is spatially uncorrelated (blue line, $c^2=0$). However, with increasing spatial correlations in the noise, the smoothing becomes less effective. Compared with resting-state data, it is clear from the figure that spatial correlations in 3T fMRI noise are significant. However, the slope of the corresponding curve is steeper than for the curves belonging to the other artificial data sets with nonzero spatial correlations of the noise. This behavior of 3T fMRI noise is due to a decrease in correlations with increasing distance. In Fig.2, we plotted the variogram [2] of 3T resting-state noise. As shown, the spatial correlation is very strong for a distance smaller than 5 pixels and is negligible beyond.



Furthermore, we constructed synthetic data using artificial activation patterns of sizes {1 to 9, 4×4, and 5×5} with random (but convex) shapes. Two inference methods were used to detect activations: Single pixel correlation analysis without and with Gaussian spatial smoothing for FWHM = {1,2,3,4} pixels. The area under the ROC curve [3] belonging to a false positive fraction of 0 to 0.1 was used as a measure of detection power. As an example, Fig.3 lists the results for artificial noise data with c^2 = {0, 0.1, 0.3, 0.5} and resampled resting-state data for FWHM=2 pixels which had the best overall performance (solid lines correspond to single pixel analysis without smoothing, broken lines to single pixel analysis with smoothing). When there is no correlation among the noise, spatial smoothing is superior for activation sizes larger than 4 pixels (blue lines). However, if the noise is spatially correlated with c^2=0.5 similar to resting-state data (black lines), spatial smoothing performs only better for large-sized activation patterns (≥4×4 pixels).



Finally, we show some activation maps from a memory paradigm (the same scanning protocol as resting-state data, total 288 time frames with first 5 discarded). The contrast is "Encoding minus Control" with the significance level at corrected $p<0.05$ [4]. We tested 3 methods: (1) GLM without smoothing, denoted as GLM-NS; (2) GLM with Gaussian smoothing (FWHM=2.24 pixels), denoted as GLM-GS; and (3) a local constrained canonical correlation analysis corresponding to an adaptive smoothing, denoted as cCCA. Using GLM-GS, activation patterns become bulgy and have some unlikely connections (see white arrow). Note also weak and localized activations in the posterior cingulate region (see black arrows) where GLM-GS failed to detect activations. The adaptive smoothing (cCCA) method detected more (homogenous) activations than GLM-NS.

Conclusion For spatially correlated noise in 3T fMRI data, spatial smoothing with fixed FWHM is far less effective. In order to optimize detection power for small and weak activations, more advanced locally adaptive smoothing kernels should be applied. Another purpose of Gaussian smoothing for utilizing random field theory [5] to correct family-wise error rate can be replaced by a non-parametric resampling approach proposed in [4].

References and Acknowledgement [1] Worsley K, et al. 1995. NeuroImage, 173-. [2] Cressie N, 1993. Wiley-Interscience. [3] Metz C. 1978. Semin Nucl Med, 283-. [4] Nandy R, Cordes D., 2007. NeuroImage, 1562-. [5] Worsley, K, et al., 1996. Hum Brain Mapp, 58-. *This work is partially supported by the NIH (1R21AG026635).*