# Stochastic Fibre Tracking: An Average Curves Approach

## N. Ratnarajah<sup>1</sup>, A. Simmons<sup>2</sup>, and A. Hojjat<sup>3</sup>

<sup>1</sup>Department of BioSciences, University of Kent, Canterbury, Kent, United Kingdom, <sup>2</sup>Centre for Neuroimaging Sciences, Kings College London, United Kingdom, <sup>3</sup>Department of BioSciences, University of Kent, United Kingdom

### Introduction

Uncertainty and its' representation have an important role to play in the fibre tractography methods to infer useful information from noisy diffusion weighted data [1]. Probabilistic fiber tracking approaches [1, 2] have received considerable interest recently for resolving orientational uncertainties. These methods have two stages. In the first stage, they model the uncertainty at each voxel using a probability density function (PDF) of fiber orientations. In the second stage, the tracking algorithm repeats a streamline propagation process many times with the propagation direction randomly sampled from the PDF of fiber orientations. The most probable path from a seed point is estimated as the fibre with the maximum probability amongst the generated paths. The main drawback of these methods is their computational time and memory requirement. We consider the random paths as a set of curves, and find the most probable path estimated directly from the generated curves using an average curves approach. Here, a simple stochastic model [3] is used to implement two methods. Their performances are compared with both phantom and an *in vivo* data.

#### Theory

A single representative curve from a given collection of curves in space is one with the minimum difference from all others. The minimum distance from a point a in a curve A to curve B is  $M(a,B) = \min_{b \in B} (|b-a|)$  and the nearest point from a to B is  $b(a) = \arg\min_{b \in B} (|b-a|)$ . The classical measure of distance between two point-sets is the Hausdorff measure. The asymmetric Hausdorff distance from curve A to curve B is given by  $H'(A,B) = \max_{a \in A} (|b(a)-a|)$ . The average minimum distance between A and B is G'(A,B) = Mean(|b(a)-a|) which can be symmetrised by taking the mean of G'(A,B) and G'(B,A).

### **Material and Methods**

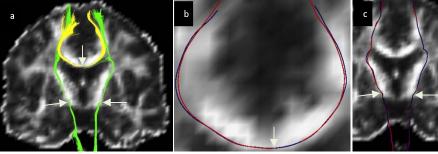
We implemented two types of average curve methods for this experiment. Method1-the average curve is selected from the collection as the curve which differs least from all other curves. We used different distance criteria to choose the best curve. Method2- the average curve is computing using an arc-length re-parameterisation method which re-parameterises the curves by placing a high number of points on each curve at equal arc length steps. Each of the points is then averaged over a number of curve-instances to produce the vertices of the average curve. A fiber path can be described as a sequence of points  $x_n \varepsilon V$ , where V is a bounded set in the 3-dimensional space. Fiber tracking algorithms determine how to obtain the sequence of points  $x_n$  for a starting point  $x_0$ . Prigarin et al suggested a stochastic tracking model [3] described by  $x_n = x_{n-1} + v_{n-1} \Delta t + \sqrt{\Delta t} \sigma \varepsilon_n$ . Here  $\varepsilon_n$  represents independent standard normal random vectors,  $v_n$  is the principal eigenvector of the point  $x_n$  and  $\Delta t$  and  $\sigma$  are step size and noise intensity parameters. If  $\sigma = 0$  the algorithm becomes deterministic. Repeating the algorithm 1000 times from a seed point with particular parameters and with the main stopping criterion of FA $\leq$ 0.15 generates two sets of curves (positive and negative directions) from a seed point. We then applied the average curves method to each set of curves and the resultant curves were concatenated to obtain the full fiber path. We used phantom data for estimating  $\sigma$  for different SNR levels of  $in\ vivo$  data and evaluated the performance of the average curves methods.

Phantom Data: The phantom images for simulating tractography errors were generated using the MRtrix software package with the following parameters: number of diffusion encoding directions = 30, b-value =  $1000 \text{ s/mm}^2$ , TE = 90 ms, NEX = 4 and resolution: $1x1x1 \text{ mm}^3$ . We performed fiber tracking with a constant step size 0.1mm and with different  $\sigma$  values for different trajectory structures (linear, kissing and spiral) with a range of SNR levels to generate the computed curve C. The true curve T is the axis of each fibre sampled. The performance measures ( $\xi$ ) were calculated using the symmetrised average minimum distance between T and C, and the Hausdorff distance from C to T.

In vivo: Diffusion-weighted data were acquired from a healthy human brain on a 1.5 Tesla scanner with an image matrix of 128x128, 60 slice locations covering the whole brain and 1.875x1.875x2.0 mm³ spatial resolution. Diffusion weighting was distributed along 41 directions using a b-value of 1000 s/mm² and there were 6 volumes with no diffusion weighting. Random trajectories were generated from a seed point in the corpus callosum and two seed points in the capsula interna with step size 0.2 mm and  $\sigma = 0.03$ , which was estimated from phantom data results with minimum error. The most probable fibre paths were obtained using the average curve methods.

# Results and Discussion

Table 1 reports mean and standard deviation of the performance measures ( $\xi$ ) from the phantom data for selected  $\sigma$  values. The result shows that the average curve methods (especially method2) are more accurate than streamline tracking with significantly lower errors at corresponding SNR levels. We also observe that the error value decreases up to some value of  $\sigma$ , especially the average minimum distance in method2 (Figure2). It would be helpful to determine  $\sigma$  as a function of the SNR of the dataset in question. Figure 1 (a) shows the random trajectories seeded from a point in the Corpus Callosum and two seed points in the Capsula interna.



The resulting tracks from the average curves method are presented in Figure 1(b) and (c). Our new methods are fast and relatively easy to implement with good tracking capabilities especially in the low SNR data. Although the initial results are very promising, further work is required with more probability models and more *in vivo* datasets to confirm our findings.

### Acknowledgements

We would like to thank Prof. Malcolm Sabin from University of Cambridge for stimulating discussions about average curves. **References** 

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Figure 1: Generated random tracks and the resultant curves (Blue-Method 1, Red-Method 2) on coronal FA images.

		StM	Method 1				Method 2			
SNR		$\sigma = 0$	0.001	0.003	0.005	0.008	0.001	0.003	0.005	0.008
5	A	<b>0.91</b> ±0.6	0.81±0.5	0.77±0.5	0.75±0.4	0.75±0.5	$0.80\pm0.6$	0.80±0.6	0.78±0.6	$0.70\pm0.4$
	Н	<b>3.79</b> ±3.4	2.82±2.6	3.35±2.9	2.56±2.5	2.80±2.6	3.52±3.1	3.19±2.8	3.32±2.6	3.33±2.4
15	A	<b>0.71</b> ±0.6	0.66±0.5	$0.59\pm0.4$	0.57±0.3	0.73±0.6	$0.80\pm0.7$	0.78±0.7	0.66±0.5	0.71±0.7
	Н	<b>2.78</b> ±2.6	2.58±2.4	2.54±2.5	2.48±2.4	2.42±2.4	2.48±1.9	2.53±2.1	3.16±2.7	3.15±2.7
30	A	0.56±0.4	0.55±0.4	0.52±0.4	0.51±0.3	0.79±0.6	0.53±0.4	0.49±0.4	0.51±0.4	0.52±0.4
	Н	<b>2.73</b> ±2.6	1.76±1.6	1.63±1.3	2.12±1.7	2.56±2.4	2.12±2.1	2.80±2.7	3.32±3.1	2.86±3.0

Table 1: The performance measures (StM – Streamline Method, A-AverageMinimumDistance, H- Hausdorff distance) Figure 2: Plot of A versus σ using the Method 2