Using a mixture of Watson distributions to estimate intrinsic fiber diffusion properties

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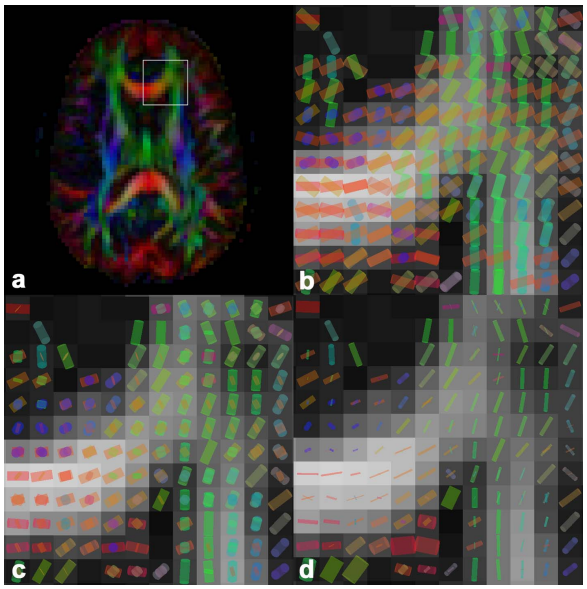
INTRODUCTION: Diffusion tensor MRI (DT-MRI) has been useful in estimating microscopic changes in tissue based on water molecular diffusion. However, DT-MRI is inappropriate in regions with non-parallel fibers [1]. To overcome this, high angular resolution diffusion (HARD) encoding schemes have been proposed [2] and used for finite mixture of Gaussians [2], q-Ball imaging [3] and spherical deconvolution (SD) [4]. However, most of the HARD imaging methods are aimed at estimating fiber orientation, and lack further analysis for estimating individual fiber properties, such as fiber diffusivity and volume fraction, within a single voxel. In this study, we derived a simple relationship between a mixture of Watson directional distributions [5], representing the diffusion weighted (DW) signal in the FORECAST SD model [6], and estimated individual fiber properties, e.g., fiber orientation, volume fraction and radial diffusivity. We accomplished this by fitting the observed DW signal to the Watson distributions of up to two fibers within a single voxel. A similar work has been presented for modeling the orientation distribution function (ODF) [7].

METHODS: The Watson distribution is described as $f(\theta;\alpha)=C(\alpha)\exp[-\alpha\cos^2\theta]$ for a three dimensional random vector on a unit sphere with normalization constant $C(\alpha)$, here $C(\alpha)=\sqrt{\alpha}/(2\pi^{3/2}erf[\sqrt{\alpha}])$ and θ is the angle between fiber axis and diffusion weighting gradient vectors. The FORECAST response function, $g(\theta;\lambda_\perp)=\exp[-b\lambda_\perp]\exp[-3b(\bar{\lambda}-\lambda_\perp)\cos^2\theta]$ with $\bar{\lambda}$ (mean diffusivity) and $\lambda_\perp$ (radial diffusivity), can be represented as a Watson distribution such as, $1/C(\alpha)\cdot\exp[-b\lambda_\perp]\cdot f(\theta;\alpha)$, where $\alpha=3b(\bar{\lambda}-\lambda_\perp)$. Then the FORECAST representation of the DW signal can be written $S=1/C(\alpha)\cdot S_0\cdot\exp[-b\lambda_\perp]\cdot\int ds\,P\cdot f(\theta;\alpha)$, where S is the DW signal, S_0 is the signal for *b*-value=0 and *P* is the fiber orientation distribution function on a unit sphere. Considering the zeroth order spherical harmonic coefficient of the FORECAST response function, $c_0=4\pi\bar{S}/S_0=1/C(\alpha)\exp[-b\lambda_\perp]$ with mean DW signal $\bar{S}$, the DW signal can be simply represented using the Watson distribution as $S/(4\pi\bar{S})=\int ds\,P\cdot f(\theta;\alpha)=\sum p_i\cdot f(\theta;\alpha_i)$ for discrete fiber bundles. This model can be used for fitting the observed DW signal with estimates of individual fiber orientation, radial diffusivity and volume fraction for each voxel. The use of one or two fibers in the fit was determined based on the correlation coefficient between observed and fitted signal. DW-MRI data were acquired on a 3T Philips scanner (Philips Medical System, Cleveland, USA) with FOV=240×240, voxel size=2.5mm isotropic, 35 slices with no gap, TR/TE = 9000/55ms, *b*-value=0 and 2000s/cm^2 with 46 diffusion weighting directions.



RESULTS: Figure 1(a) shows a color-coded fractional anisotropy (FA) map with ROI (red = right/left, green = anterior/posterior, and blue = inferior/superior fiber orientation). Figures 1(b)-1(d) show individual fiber orientations for each voxel represented by a cylinder. Individual fiber properties are presented with cylinders of equal length and width in (Fig.1b), with cylinder height proportional to fiber volume fraction (Fig.1c), and with cylinder radius proportional to radial diffusivity (1d) .

DISCUSSION: It is clear that in most voxels, one fiber component is dominant over the other component, based on the volume fraction and radial diffusivity (note that smaller radial diffusivity is equivalent to larger fiber FA). More robust estimation of the number of fiber components will likely improve the accuracy of the analysis.

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