Regularized Q-ball Reconstruction: Robust estimation, model selection, and spatial denoising

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Introduction

Diffusion weighted imaging (DWI) with high angular resolution allows in principle the detection of complex microstructure such as the crossing of white matter fiber tracts. This task requires a model connecting DWI signal with a distribution describing the random motion of water molecules. Q-ball [1] is a model designed with this aim. It captures the orientation distribution function (ODF) defined over the unit sphere, that represents the probability of diffusing in a given direction. In WM, it has been shown that the peaks of ODF correspond to the presence of fiber tracts passing through the voxel. Q-ball model can be implemented using combinations of spherical harmonics [2,3], this requires the estimation of large numbers of degrees of freedom. Different ideas have been tried to make the estimation more robust, such as Tikhonov regularization or denoising [4]. Here we propose an implementation of Q-ball that estimates only a few meaningful coefficients and reduces spatial incoherence. In order to evaluate the robustness of the method we used a dataset from post mortem brain tissue.

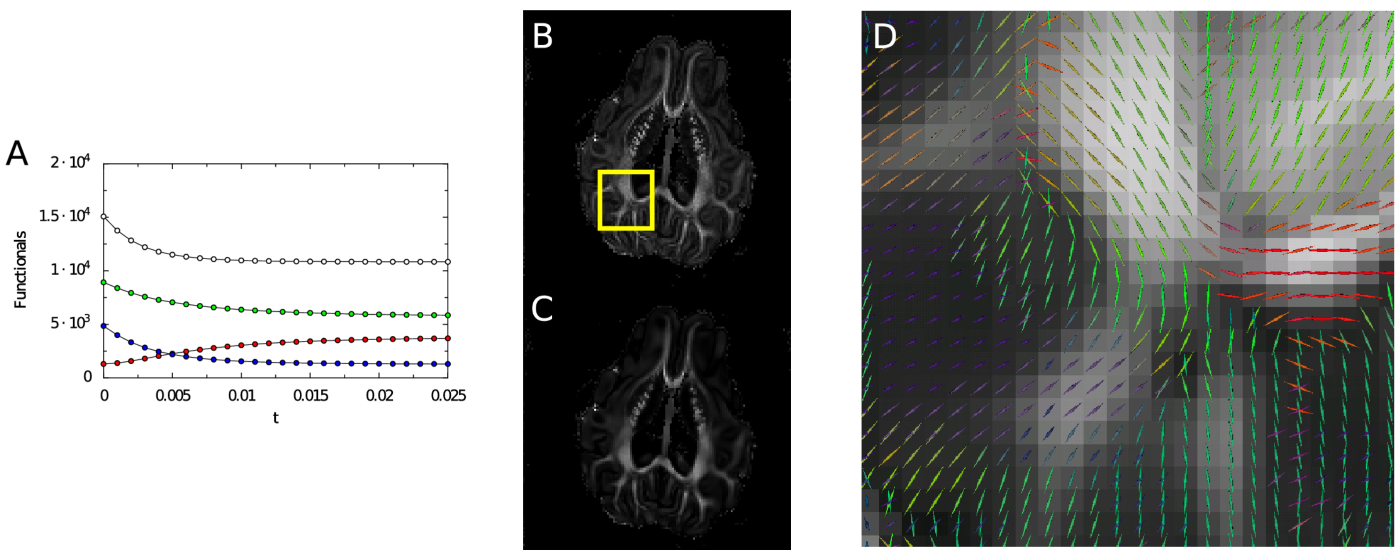
Methods

The Q-ball model is implemented in the following way: at each voxel the ODF is described as a linear combination of real spherical harmonics (RSH), the real coefficients $c(l,m)$, for every degree $l=0,2,4,...$ and order $m=-l,-l+1,...,l-1,l$, being the unknown parameters that we wish to estimate for every voxel. The coefficients are found by minimizing an energy functional made up of three terms: (1) a fidelity term that is the log-likelihood of the Rician distribution [5] and compares the acquired signal with the signal reconstructed fom the coefficients; (2) a spatial smoothness term that penalizes the differences between the coefficients at one voxel and the coefficients at a neighboring voxel, following the Total Variation method [6], this choice should reduce noise, preserving edges; and (3) a sparsity (angular smoothness) term that penalizes nonzero coefficients and respects rotational invariance (thus avoiding orientation bias), combining the 1-norm for the contribution of each degree with the 2-norm of the coefficients of the same degree. In simple words, the last term selects the number of degrees of freedom for each voxel. The proposed functional is convex and the minimization can be performed by a gradient descent algorithm that converges to the unique minimizer [7] i.e. the best set of coefficients that describe the ODF in the RSH basis.

The acquisition uses a spherical scheme with 61 unique gradient directions. Diffusion weighted imaging was obtained in a perfusion fixated pig brain on an experimental 4.7T Varian Inova scanner (see [8] for a complete description). The diffusion-weighting gradient strength G = 61 mT/m; pulse onset separation DELTA = 30 ms; pulse width delta = 23 ms; TE = 60 ms; b= 3146 s/mm2; NEX=2. The image has 10 slices with in-plane resolution 128x128; voxel size 0.5x0.5x0.5 mm3.

Results

For the pig brain dataset, the proposed method was able to find the coefficients and the contribution of each degree of the RSH expansion. The process took around 5 min per axial slice in a Pentium IV 1.8 Ghz computer. Fig. 1A shows the convergence of the gradient flow as the algorithm iterated: the residuals (red dots) increased as the sparsity (green) and smoothness (blue) penalizations become smaller; as a result the total energy (white) is reduced. After convergence, fewer coefficients are used in most voxels. Fig. 1BC shows the contribution of degree $l=2$, before (B) and after (C) the regularization. The regularized image shows higher spatial coherence and better delineation of highly anisotropic white matter regions. Some small structures, that are not well supported by data, were removed by the regularization. Fig. 1D shows the orientations of the peaks of the ODFs in the region indicated in Fig. 1B by a yellow rectangle.



Discussion

We demonstrate a variational approach that computes ODFs from high angular resolution data, using a RSH expansion computed as the minimizer of a functional that combines fidelity to the acquisition, sparsity of degrees of freedom, and spatial smoothness. The whole approach respects rotational invariance, doesn't introduce orientation bias and selects a few nonzero coefficients. The peaks of the reconstructed ODF can be used to detect fiber crossings. The multiple fiber orientation found by this method could be used by tractography algorithms for better connectivity maps and tract delineation in complex regions.

References

[1] D.S. Tuch, MRM 52, 1358–1372, 2004
[2] C.P. Hess et al. MRM 56:104–117, 2006
[3] M. Descoteaux et al. MRM 58(3):497–510, 2007.
[4] H.E. Assemlal, D. Tschumperle and L. Brun. Technical report, GREYC, 2007.
[5] S. Basu, T. Fletcher, and R. Whitaker. MICCAI, 9, 117–25, 2006.
[6] T.F. Chan, S. Osher, and J. Shen. IEEE TIP 10(2):231–241, 2001.
[7] S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge, 2004
[8] T.B. Dyrby et al. Neuroimage 37, 4, 2007