Diffusion Propagator Imaging (DPI): an alternative to Diffusion Spectrum Imaging (DSI)

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INTRODUCTION: The quest of diffusion-weighted (DW) imaging is to non-invasively obtain information about the average diffusion of water molecules in biological tissue. Many recent high angular resolution diffusion imaging (HARDI) [1, and references therein] techniques are proposed to infer the diffusion or fiber orientation distribution function (ODF), but this ODF only captures the angular structure of the diffusion process. In this work, we are interested in the reconstruction of the full three-dimensional (3D) ensemble average propagator (EAP) describing the diffusion process. Under the narrow pulse assumption, the relationship between the diffusion signal attenuation, E(**q**), in q-space and the EAP in real space, P(**R**), is given by an Fourier transform (FT), P(**R**) = FT[E(**q**)]. Diffusion Spectrum Imaging (DSI) [2] is currently the only established method exploiting this relation in order to reconstruct a model-free EAP in the human brain. DSI measures DW images along as many directions and as many q-values as possible on a 3D Cartesian grid before computing the FT giving the EAP. More recently, another technique was proposed to perform measurements along many radial lines before computing 1D tomographic projections to reconstruct the 3D EAP [3]. Other techniques suggest using multiple spherical shells sampling, but reconstruct other functions than the EAP, such as generalized high order tensors [4] or the diffusion orientation transform (DOT) [5] or the Kurtosis [6] or a better diffusion ODF [7]. As of today, most methods for EAP reconstruction ([2,3]) unfortunately need to use more than 200 DW measurements, considerably limiting their application. We present diffusion propagator imaging (DPI), a novel technique for simple and linear analytical EAP reconstruction using Laplace's equation. We show that EAP reconstruction from DPI is similar to the established DSI reconstruction from the same subject, and is, thus, an alternative.

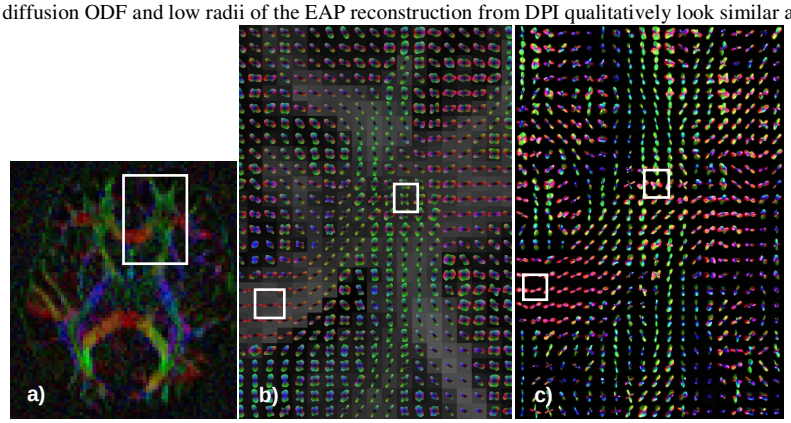
METHODS: If we suppose that Laplace's equation can describe the 3D q-space MR diffusion signal, $E(\mathbf{q}) = S(\mathbf{q}) / S_0$, can be written as,

$$E(\mathbf{q}) = E(q\mathbf{u}) = \sum_{j=0}^{\infty}\left[\frac{c_j}{q^{l(j)+1}} + d_j \cdot q^{l(j)}\right] Y_j(\mathbf{u}), \text{ from which the EAP is given by, } P(R_0\mathbf{r}) = \sqrt{\frac{8\pi}{R_0}}\sum_{j=0}^{\infty}\frac{(-1)^{l(j)/2}(2\pi R_0)^{l(j)-3/2}}{(2l(j)-1)!!}\cdot c_j Y_j(\mathbf{r}), \quad \text{where } (n-1)!! = (n-1)\cdot(n-3)\cdot\ldots\cdot 3\cdot 1,$$

q is a 3D vector in q-space, q = |**q**|, **u** and **r** is a 3D unit vector, Y_j is the modified even, symmetric, real, and orthogonal spherical harmonic (SH) basis [1], l(j) is the order associated with the jth element of the SH basis, and $P(R_0\mathbf{r})$ represents the probability of finding a water molecule at distance $R_0\mathbf{r}$ for the origin. Hence, it can be viewed as the EAP values on a sphere of radius R_0. Good boundary conditions need to be given to solve for the SH coefficients, due to the Laplace equation assumption. In our problem, we need the signal without diffusion gradient S_0 (when q = 0) and at least two different b-value shell measurements. Intuitively, this can be thought as the heat equation between each shell. A priori, there is no physical reason why this assumption should be able to describe the q-space signal but Laplace's equation can model the diffusion signal satisfactorily and allow one to solve for the EAP very efficiently. Note that the EAP, $P(R_0\mathbf{r})$, only depends on the c_j coefficients; the d_j coefficients have disappeared in the EAP reconstruction. However, these d_j are needed for accurate E(**q**) modeling. In the end, the EAP solution has an analytical form, is linear and compressed because represented by only 15 coefficients, if one uses a SH order 4 for the signal estimation.

To validate and compare the proposed method, DPI and DSI were done on the same healthy volunteer. For DPI, the data was acquired on 3T Trio MR system with TE/TR=147ms/11,5s, BW=1680Hz/pixel, 96x96 matrix, isotropic 2mm resolution, and 60 axial slices. A non-weighted (b=0) diffusion image followed by two b-values acquisitions with 64 uniform directions, at b = 1000 s/mm^2 and b = 6000 s/mm^2, were done. For DSI, the same 3T system was used with 2mm isotropic spatial resolution, and, as in [2] and [8], 515 DW measurements were acquired comprising in q-space the points of a cubic lattice within the sphere of five lattice units in radius (see [2],[8]), maximum gradient amplitude of 40 mT/m, and b_{min} = 0 to b_{max} = 6000 s/mm^2, resulting in a q-space isotropic resolution of $R_{0min} = q_{max}^{-1}$ = 14.1 µm and $R_{0max} = q_{min}^{-1}$ = 42.9 µm. DSI is normally used to obtain the angular structure of the spectrum, the ODF, defined as $\text{ODF}(\mathbf{r}) = \int_a^b P(R_0\mathbf{r})R_0^2 dR_0$. As for DPI, we also compute $P(R_0\mathbf{r})$ for different R_0 values with tri-linear interpolation from the discrete propagator obtained from the Fourier transform.

RESULTS: Fig a) shows the region of interest from axial RGB, b) the ODF computed from DSI, and c) the EAP for R_0=2µm. Then, the table illustrates EAP from both DSI and DPI, from isolated voxels containing single fiber and crossing fiber configurations respectively (in boxes in Fig. b)-c)). Reconstructions from both the DSI diffusion ODF and low radii of the EAP reconstruction from DPI qualitatively look similar and agree with the underlying anatomy. Note that EAP reconstruction from DSI is more vulnerable to noise as the radius of the EAP estimation increases and reaches the boundary of q-space / real space. This is less the case for DPI.



Single fiber EAP reconstructions for different radius R_0							
R_0(µm)	18.2	22.3	26.4	30.5	34.6	38.7	42.8
DSI							
DPI							
Crossing fibers EAP reconstructions for different radius R_0							
R_0(µm)	18.2	22.3	26.4	30.5	34.6	38.7	42.8
DSI							
DPI							

DISCUSSION: This work shows that with DPI, the average diffusion propagator (EAP) can be reconstructed robustly from only two spherical shells with less DW measurements than a standard DSI acquisition. The advantages of DPI are threefold: 1) The solution is analytical and thus allows one to reconstruct the EAP for radii values outside the q-space resolution prescribed by $q_{max} = 1/R_{0min}$ and $q_{min} = 1/R_{0max}$. 2) Signal estimation with Laplace's equation performs some level of smoothing, especially needed for high b-value measurements. In DSI, as seen in the table, the EAP is often very noisy near the boundaries of real-space, and thus, DSI also does need some smoothing in practice. First, a Hanning window is used to premultiply the signal before Fourier transform computation and second, the ODF is most often computed for boundary limits (a, b in above integral) that only cover some middle portion of the diffusion spectrum. No such 'ad hoc' low-pass and high-pass filtering is needed in DPI. 3) The solution is linear and compact, i.e. expressed in a small number of coefficients. Here, DPI results shown used an order 4 SH basis solution, which requires 30 coefficients to estimate the diffusion signal and only 15 to reconstruct the EAP. Hence, it is likely that less than 64 directions on each shell can be enough to reconstruct the EAP. Phantom results seem to support this claim. Therefore, DPI with less than 100 DW measurements seems promising on the human brain.

References: [1] Descoteaux, PhD Thesis, 2008. [2] Wedeen et al, MRM, 2005. [3] Pickalov & Basser, ISBI, 2007. [4] Liu et al MRM, 2004. [5] Ozarslan et al, NeuroImage, 2006. [6] Lu et al, NMR Biomed, 2006. [7] Khachaturian et al, MRM 2007. [8] Lin et al, NeuroImage 2003.

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