A COMPARISON OF TWO MODELS OF ANOMALOUS DWI BASED ON A KNOWN DISTRIBUTION OF WATER DIFFUSION RATES

C-Y. LEE[1], K. M. Bennett[2], L. J. Karam[1], and J. P. Debbins[3]

[1]Department of Electrical Engineering, Arizona State University, Tempe, AZ, United States, [2]Harrington Department of Bioengineering, Arizona State University, Tempe, AZ, United States, [3]Keller Center for Imaging Innovation, Barrow Neurological Institute, Phoenix, AZ, United States

Introduction: Diffusion-weighted imaging (DWI) signal attenuation in the brain is non-monoexponential [1,2]. Understanding the source of non-monoexponential behavior may lead to an improved characterization of the tissue during disease. Several phenomenological models have been developed to describe this data. Two such models are stretched exponential (αDWI) [3] and second-order cumulant [6] fits (DKI), which use the same number of fitted parameters but have very different theoretical underpinnings. Both α and K_{app} are thought to reflect heterogeneity in water diffusion rates [3-7], but their relationship to the underlying biophysics is still unclear. In this work, we simulated a simplified physical system of either one or two water compartments with a known distribution of water diffusion rates, and studied how the stretched-exponential and cumulant expansion models behaved when the mean and variance of the distribution were changed. This work gives an improved understanding of the sensitivity of each model to the underlying physics, and has practical implications for the choice of model for a given DWI experiment.

Method: A computer simulation was created in Microsoft Visual C++. 30,000 spins were placed randomly in a three-dimensional volume and moved independently in a 3D random walk. The step size of each spin was obtained from a Gaussian distribution generated using Box-Muller transform [8]. In the two-compartmental model, the step sizes of the spins were sampled from two Gaussian distribution. Fig. 1 shows a simulated one-compartment model with decreasing mean (E[D]) and increasing variance (var[D]) of diffusion rates, and a two-compartment model with decreasing volume fractions of fast diffusion compartment (V_f). A simulated PGSE sequence with $\delta \approx \Delta \approx 10$ ms was applied along x-axis. The signal relaxation was produced by adding the phase shift of each spin through the applied field gradient. Three b-value ranges were set to be: 0-2500, 0-5000, and 0-7500 s/mm^2, satisfying $b \times E[D] \geq 1$, $b \times E[D] \geq 2$, and $b \times E[D] \geq 3$, in increments of 250 s/mm^2 by increasing gradient strength g. Stretched exponential and second-order cumulant models were fitted to the data using the Levenberg-Marquardt algorithm in MATLAB (Mathworks, Inc.). Data below the noise floor were excluded from the data fitting.

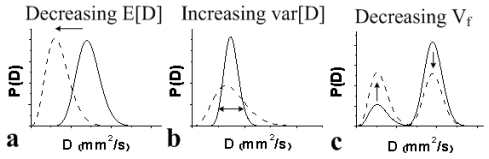

*Fig. 1: Simulated one- (**a** and **b**) and two-compartment (**c**) diffusion models, showing the distribution of water diffusion rates and how they were varied in the experiment (arrows).*

Results: The sensitivities of α to both changes in E[D] and var[D] increased as b-value range increased, and α was relatively sensitive to the changes of var[D], especially at high kurtosis over three ranges of b-values (Fig. 2). Unlike α, the sensitivity of K_{app} to changes in E[D] and var[D] decreased as the b-value range increased. The sensitivity of K_{app} to changes of E[D] were higher over three ranges of b-values, but sensitivities was lower when kurtosis was high, at b-value ranges above 5000 mm^2/s (Fig. 3). In the two-compartment model with decreasing V_f, E[D] and var[D] were altered meanwhile (Fig. 4a). α only tracked the changes of var[D], especially at b-value ranges above 5000 s/mm^2 (Fig. 4b), consistent with the previous observations in Fig. 2. K_{app} correlated poorly with kurtosis above 1.3, and its values apparently saturated with increasing b-value ranges (Fig. 4c). Both DDC and D_{app} tracked the changes of E[D], and the effects of b-value ranges were not significant (Fig. 4b and 4c).

Discussion and Conclusions: Our results showed that α was sensitive to changes in diffusion heterogeneity and tracked changes well over three b-value ranges. Its sensitivity to diffusion rate increased with increasing b-value ranges. K_{app} basically correlated with kurtosis, but only when b was low. This might be due to the effects from truncating cumulant expansion. In conclusion, α is directly related to var[D] in both the one- and two-compartment models. K_{app} is sensitive to both to E[D] and var[D], which is expected from the model definition. αDWI may be preferred when high b-values can be used, but DKI may be preferred at low b-values. However, because DKI is limited to low b-value ranges, it may be insensitive to changes in slow diffusion. These two models are thus similar, but have separate relationships to the intravoxel distribution of water diffusion rates.

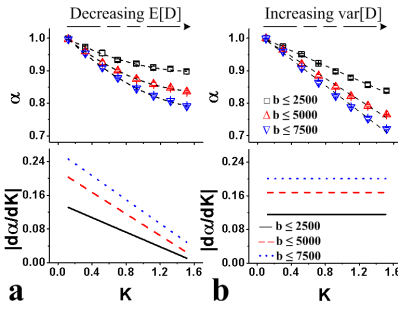

*Fig. 2: α was more sensitive to the changes of var[D] (**b**) than those of E[D] (a), and tracked changes of var[D] over three b-value ranges.*

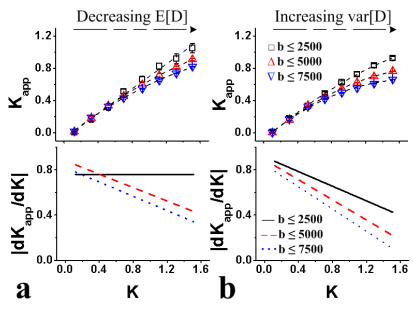

*Fig. 3: K_{app} was more sensitive to the changes of E[D] (**a**) than those of var[D] (**b**), and only tracked changes of E[D] at b-value range: 2500 s/mm^2.*

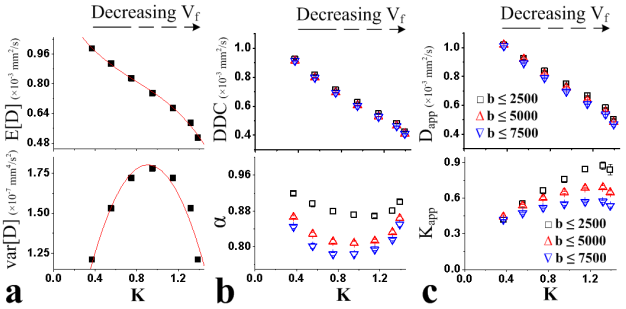

*Fig. 4: E[D] and var[D] were varied meanwhile (**a**). Both DDC (**b**) and D_{app} (**c**) tracked the changes of E[D]. α (**b**) only tracked the changes of var[D]. K_{app} (**c**) generally correlated with kurtosis, except for the kurtosis above 1.3.*

Reference: [1] Pfeuffer J et al. MAGMA. 8(2), 1999. [2] Mulkern RV et al. MRM. 44(2), 2000. [3] Bennett KM et al. MRM. 50(4), 2003. [4] Magin RL et al. JMR. 190(2), 2008. [5] Hall MG et al. MRM. 59(3), 2008. [6] Jensen JH et al. MRM. 53(6), 2005. [7] Lu H et al. NMR Biomed. 19(2), 2006. [8] Papoulis A et al., Random Variables and Stochastic Processes, McGraw-Hill, New York, 1999.