Groupwise non-rigid registration of 4th-order diffusion tensor fields and atlas construction

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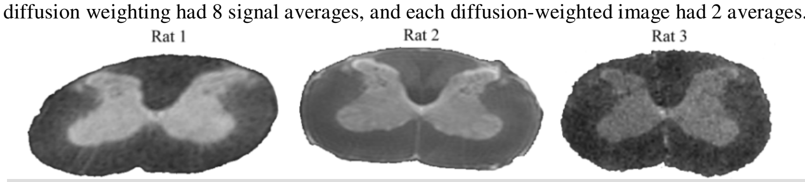
Introduction: The local diffusivity function in a diffusion-weighted MRI (DW-MRI) dataset can be approximated by a higher-order tensor, which can capture complex tissue structures such as fiber-crossings [1]. A 4th-order tensor can be expressed in the standard notation of ternary forms as: $D(\mathbf{g}) = \sum_{i,j,k=4} d_{i,j,k} g_1^i g_2^j g_3^k$ where $d_{i,j,k}$ are the 15 unique tensor coefficients, and $\mathbf{g} = [g_1\ g_2\ g_3]^T$ is the magnetic field gradient direction. A field of positive 4th-order tensors can be estimated from a DW-MRI dataset by using the method presented in [2]. Registration of estimated tensor fields is problem encountered in many tasks e.g., in atlas construction. Here, we should emphasize that tensor field registration cannot be achieved by employing scalar-valued image registration techniques in a component-wise fashion [3]. Two 4th-order tensor fields can be registered to each other by using locally-affine transformations [3], however, the results produced by such an algorithm are inaccurate and may contain artifacts due to the discontinuities caused by the extent of the local support of the transformation parameters. In this work we present a method for simultaneous groupwise non-rigid registration of 4^{th} order tensor fields and atlas construction. This is achieved through a non-trivial generalization of the approach presented in [4]. To achieve the task of registration, we first need a distance measure on 4^{th} order tensors that can be used to match them.

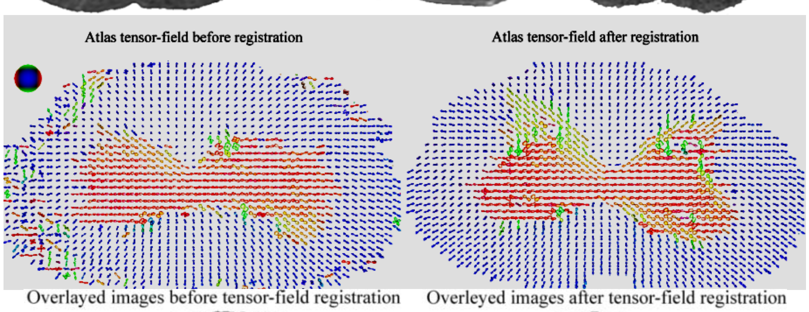
Distance measure: Since both the DW-MRI signal response and the estimated 4th-order tensors are positive-valued functions, we employed the natural Riemannian metric of positive real umbers given by $\|a, b\|_{R+} = |\log(a/b)|$. This metric can be extended to the case of positive-valued functions f and h by integrating over their domain as follows: $dist^2(f(\mathbf{g}), h(\mathbf{g})) = \int \|f(\mathbf{g}), h(\mathbf{g})\|_{R+}^2 d\mathbf{g}$.. By setting the functions f and h to be the DW-MRI images $I_1 = S_0 e^{-bD_1(\mathbf{g})}$ and $I_2 = S'_0 e^{-bD_2(\mathbf{g})}$ approximated by 4th-order diffusion tensors $D_i(\mathbf{g})$ in the Stejskal-Tanner equation, their distance $dist^2(I_1, I_2)$ can be computed analytically.

Groupwise Registration: The groupwise registration problem is now stated as follows: given M diffusion tensor fields we need to estimate deformation fields $\varphi_1, \cdots, \varphi_M$ that can transform the given fields so that the inter-data distance is minimal. In order to estimate the tensor-field atlas I_A simultaneously with the deformable registration parameters, the above problem is posed as a minimization of the distance of each given tensor-field from the estimated atlas. The problem can be solved by minimizing the energy function $E_\Omega(\varphi_1, \cdots, \varphi_M) = \int_\Omega \sum_{i=1}^M dist^2\left(I_i(x),\ \varphi_i^{-1} \circ I_A(\varphi_i^{-1} \circ x)\right) dx$, where the integration is over the 3D spatial domain, and $\varphi \circ I$ denotes the 4th-order tensor re-orientation operation [3]. The atlas is estimated at each step of the registration procedure as: $I_A(x) = argmin_I \sum_{i=1}^M dist^2\left(I, \varphi_i \circ I_i(\varphi_i \circ x)\right)$, which can be performed efficiently due to analytic computations. Finally, the registration and atlas construction is achieved by iterating until convergence the two steps: 1) estimate I_A given $\varphi_1, \cdots, \varphi_M$, and 2) estimate $\varphi_1, \cdots, \varphi_M$ given I_A.

Implementation: The energy function is minimized by evolving each deformation φ_i using a field of forces computed as follows: $F(x) = \nabla_{\varphi_i(x)} = \frac{E_{N(x)}(\varphi_1, \cdots, \varphi_i + \Delta\varphi(x), \cdots) - E_{N(x)}(\varphi_1, \cdots, \varphi_i, \cdots)}{\Delta\varphi(x)}$, where $N(x)$ is a neighborhood centered at voxel x and $\Delta\varphi(x)$ is a small velocity vector applied to the deformation field φ_i at location x. $E_{N(x)}$ needs to be evaluated 3+1 times per voxel per iteration, i.e. one for the evaluation of $E_{N(x)}(\varphi_1, \cdots, \varphi_i, \cdots)$and 3 for the evaluation of $E_{N(x)}(\varphi_1, \cdots, \varphi_i + \Delta\varphi(x), \cdots)$ for each of the three dimensions of the small step $\Delta\varphi(x)$. The complexity of $E_{N(x)}$ is dominated mostly by the tensor re-orientation step which includes the polar decomposition of the Jacobian at voxel x. Hence, the total complexity for computing the gradient field per iteration is *O(NM4c)*, where c is a constant corresponding to the time complexity of $E_{N(x)}$. Finally for the update of the deformation fields we perform computations in the frequency domain similar to that in [3], which add an *O(MNlogN)* term to the total complexity.

Experiments: In our experiments we used three diffusion-weighted MR datasets from excised rat spinal cords. The protocol used included acquisition of 22 images using a pulsed gradient spin echo sequence with TR = 1.5 s, TE = 27.2 ms, bandwidth = 30 kHz, FOV = 4.3 x 4.3 mm. After the first image set was collected without diffusion weighting ($b \approx 0$ s/mm^2), 21 diffusion-weighted image sets with G = 664 mT/m, δ = 1.5 ms, Δ = 17.5 ms and T_δ = 17 ms were collected. The image without diffusion weighting had 8 signal averages, and each diffusion-weighted image had 2 averages.





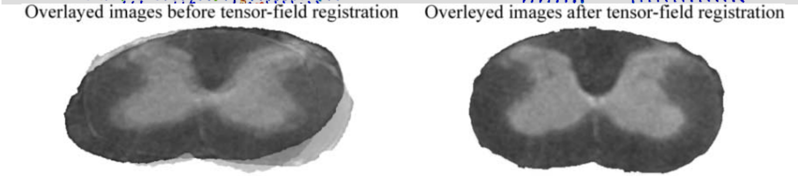


Figure 1: The S0 images from the three datasets are shown in this figure. The images correspond to an arbitrarily selected slice from the 3D data volume.

From each dataset we estimated a 4th-order diffusion tensor field by employing the method in [2].

Figure 2: ***Left:*** The 4th-order tensor field atlas computed without registering the datasets. The depicted tensor field is not coherent and the boundary between white and gray matter is rather fuzzy.
Right: The computed atlas after application of the proposed 4th-order tensor-field registration method. By visually inspecting the obtained results, we can see that the tensors have been successfully re-oriented both in the regions of the white and gray matter.

Figure 3: For visual evaluation of the result produced by the proposed registration method, we applied the estimated deformation fields to the corresponding S0 images, and then we overlaid the images using a semi-transparent mode. The result is shown on the right of this figure. For comparison we present on the left of the same figure, the original misaligned S0 images before applying the tensor-field registration.

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