Quantification of callosal widths using Conformal Mapping: Application to Multiple Sclerosis

M. P. Sampat[1], P. S. Jesrani[1], D. Meier[1], A. M. Berger[1], B. Healy[1], P. Hildenbrand[2], H. Weiner[1], and C. R. Guttmann[1]

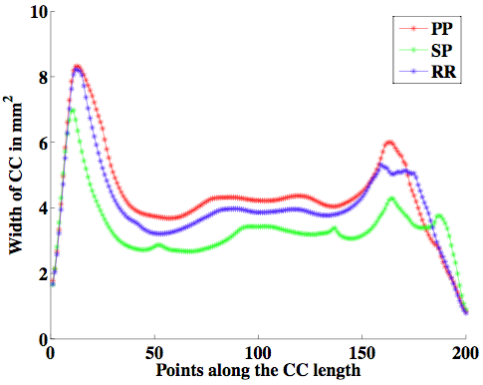
[1]Radiology, Brigham and Women's Hospital, Boston, MA, United States, [2]Radiology, Lahey Clinic, Burlington, MA, United States

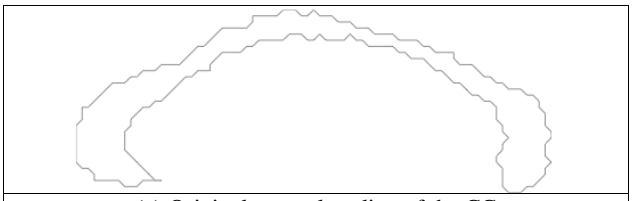
Objective: To measure the width of the corpus callosum (CC) in mid-sagittal brain MR images and to assess differences between clinical subtypes of MS.

Background: Quantitative measurements of brain atrophy in multiple sclerosis (MS) patients have shown to be of significant clinical relevance. Regional atrophy of the CC depends on the type and severity of the disease. In previous work, we found that the cross-sectional area of the CC is significantly smaller in Secondary Progressive (SP) MS patients as compared to Relapsing-Remitting (RR) or Primary Progressive (PP) MS patients. In this work, we propose a new approach to measure the width of the corpus callosum at equidistant points along the midline using a type of conformal mapping know as the Schwarz-Christoffel Mapping [1].

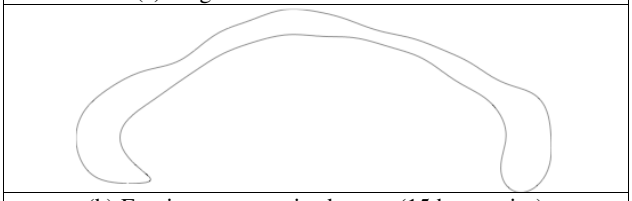
Methods: A set of 63 MS patients (29 RR, 14 SP, and 20 PP) was selected from the Comprehensive Longitudinal Investigation of MS at the Brigham and Women's Hospital (CLIMB) study. The CC was manually outlined on sagittal T1-weighted images (Figure 1a). We have developed a novel approach to measure the width of the CC. This method consists of two steps. First, we applied a Fourier parameterization [2] to the CC outline. Each coordinate pair in the image was treated as a complex number, thus reducing the 2-D problem to 1-D. The Fourier descriptors were defined by the Discrete Fourier Transform (DFT). The original boundary can be obtained by applying the inverse DFT. Using only the first P coefficients (P<N) for reconstruction implements an effective and robust form of shape smoothing. An inverse DFT with15 harmonics was implemented to give a smoothed boundary of the CC (Figure 1b).

In the second step, the smooth boundary of the CC is input to a Schwarz-Christoffel Mapping (SCM) [1]. This conformal mapping method provides an elegant solution to the problem of measuring the CC width. The SCM method is a transformation from complex analysis, which maps the upper half complex plane conformally to a polygon. In this work, we compute the SCM by using the "Cross-ratios of the Delaunay triangulation" (CRDT) algorithm proposed by Driscoll and Vavasis [1]. Thos algorithm is based on cross-ratios of the prevertices and on cross-ratios of quadrilaterals in a Delaunay triangulation of the polygon [1]. Put simply, this method creates a one-to-one mapping of information from a complex polygon (CC) to a geometrically simpler shape (e.g. rectilinear polygon), which is easier to analyze. As the SCM is a type of conformal mapping, it preserves the orthogonality of intersecting co-ordinate lines. Figure 1(c) shows the results after the SCM mapping.

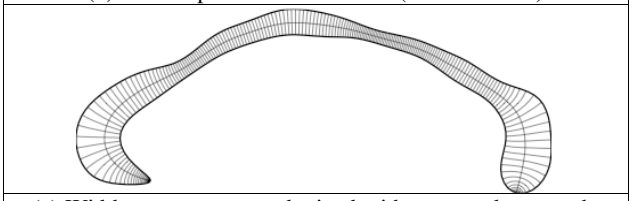




(a) Original manual outline of the CC



(b) Fourier parameterized curve (15 harmonics)



(c) Width measurements obtained with proposed approach.

Figure 1: Steps involved in the measurement of the callosal width

Results: The mean overall width's for the PP, SP and RR patients were 4.40 mm^2, 3.36mm^2 and 4.08mm^2 respectively. The overall mean width for the SP patients was significantly lesser than that for the PPMS patients ($p < 0.001$) and RRMS patients ($p = 0.01$). The widths of the RRMS and PPMS patients were not significantly different ($p = 0.18$).The mean width profile for the PPMS, SPMS and RRMS patients is shown in Figure 2. We observe that the width of the CC is the least for the SPMS patients, which indicates greater damage to the CC in SPMS. The degeneration along the length of the CC is observed to be non-uniform.

Discussion: The degeneration in the CC width was observed to be greater in the mid-body of the CC. Consistent with this finding, a histo-pathological post-mortem study by Evangalou and co-workers reported reductions in area, axonal density, and number of axons in the CC in MS patients compared to age-matched healthy controls [3]. In addition they observed non-uniform atrophy in the CC and reported relatively more atrophy in the 'mid-body' of the CC [3]. Thus, CC width can potentially be used in monitoring disease progression in MS.

References:

1. T. A. Driscoll and S. A. Vavasis. SIAM Sci. Comp. 19 (1998), 1783–1803.
2. Staib and Duncan et al. IEEE-PAMI, 1992.
3. N. Evangelou, M.M. Esiri, S. Smith, et al. Annals of Neurology, **47**(3), 2000, 391-395.