Quantitative susceptibility imaging with homotopic L0 minimization programming: preliminary study of brain

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INTRODUCTION

Susceptibility-weighted imaging (SWI) technique is used for neuroimaging to improve visibility of iron deposits, veins, and hemorrhage [1]. Quantitative susceptibility imaging (QSI) improves upon SWI by measuring iron in tissues, which can be useful for molecular/cellular imaging to analyze brain function, diagnose neurological diseases, and quantify contrast agent concentrations. Susceptibility quantification can be achieved by inverting local magnetic field to magnetization source. The ill-posedness of this inversion can be resolved with regularization. L_1 norm regularization minimization has shown good susceptibility estimation assuming a sparse susceptibility distribution [2]. However, L_0 norm is the optimum for sparse distributions. Although L_0 norm minimization programming is NP-hard, homotopic L_0 norm minimization can be used to approach a solution that approximates L_0 solution. In this study QSI of brain with hemorrhage was derived with homotopic L_0 norm minimization programming.

MATERIALS AND METHODS

The forward problem from susceptibility spatial distribution $\chi(r)$ to the measured magnetic field (relative field shift $b(r) = (B(r) - B_0)/ B_0$) is: $C\chi = b$, where $C(x)$ is a matrix representing the convolution kernel $[3\cos^2(\theta) - 1]/4\pi|r|^3$. The regularization solution to the inverse problem is formulated as: $\min_\chi \lambda |C\chi - b|_2^2 + |G\chi|_L$, where λ is the regularization parameter, G denotes an operator such as the gradient operator (the gradient promotes sparsity), and L denotes choices of norms: L_2, L_1 or L_0. L_0 norm is defines as $|x|_0 = \Sigma\, 1\ (x\neq 0)$ and it can be approximated by a function with a parameter tuned close to zero homotopically, i.e. $\lim_{\sigma\to 0} \Sigma\, \rho(|x|, \sigma) \approx |x|_0$. In this work $\rho(|x|, \sigma)=1-e^{-|x|/\sigma}$ was used as the homotopic approximation of L_0-norm. The homotopic L_0 minimization problem is formatted as $\min_\chi E(\chi, \sigma, \lambda) = \lambda |C\chi-b|_2^2 + \Sigma\, \rho(|G\chi|, \sigma)$ [3]. And $\Delta E = L(\chi,\sigma,\lambda) = 2A^H(A\chi-b) + \lambda G^H\Lambda(\chi, \sigma)Gx$, where $(\cdot)^H$ denotes conjugate transpose and $\Lambda(\chi,\sigma)$ is a diagonal matrix, whose components are $\rho'(|G\chi|, \sigma)/|G\chi|$. Here a weak derivative is used, e.g., $\rho(|x|) = |x|$, $\rho'(|x|) = \text{sign}(x)$. The solution of $\Delta E = 0$ is $(2A^HA + \lambda G^H\Lambda(\chi,\sigma)G)\chi = 2A^Hb$, which can be solved with conjugate gradient method. The tuning parameter σ is reduced by a factor (<1) during iterations. If $\rho(|x|) = |x|$, L_0 problem can be generalized to L_1 problem and it is more efficient than L_1 problem solved using log-barrier method. If $\rho(|x|) = |x|_2^2$, the problem becomes an L_2 problem.

Gd phantom, healthy subjects (n=5) and patients (n=3) with brain hemorrhage were imaged with a 3D axial gradient echo sequence, FOV=24 cm, slice thickness= 2 mm, image matrix=512x384, BW = 64KHz, TR/TE =51/38 ms, FA=20°. Phase was corrected with phase unwrapping and background phases were removed using high pass filtering. Homotopic L_0 minimization was used for deriving the quantitative susceptibility images. σ was initialized with 0.5 and reduced by a factor of 0.5 during iterations of the conjugate gradient algorithm.

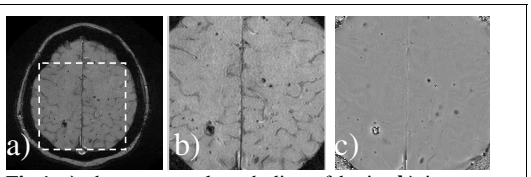

Fig.1 a) shows one selected slice of brain, **b)** is a crop from **a)**, **c)** is the corrected phase images.

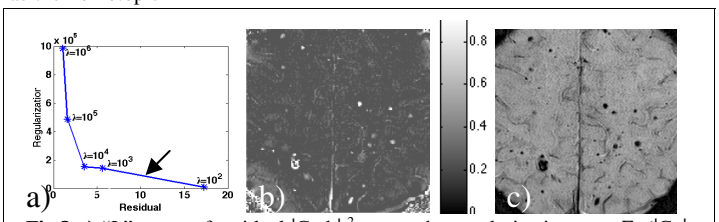

Fig.2 a) "L" curve of residual $|C\chi-b|_2^2$ versus the regularization term $\Sigma\rho(|G\chi|, \sigma)$. QSI with $\lambda=10^3$ was chosen, shown in **b)** (positive values shown), **c)** SWI.

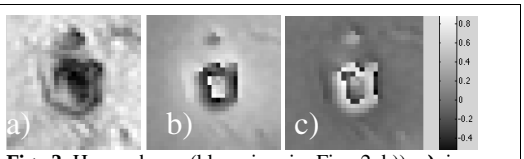

Fig. 3 Hemorrhage (blooming in Fig. 2 b)) **a)** image intensity, **b)** image phase, **c)** QSI (full scale).

RESULTS AND DISCUSSION

Fig. 1 shows the brain magnitude and phase images in patient with multiple hemorrhages. QSI solutions with various regularization parameters have been derived. An "L" shape curve of residual $|C\chi-b|_2^2$ versus the regularization term $\Sigma\rho(|G\chi|, \sigma)$ demonstrates a trade-off between data fidelity and solution sparsity, shown in Fig. 2 a). As regularization parameter λ increases, solution has smaller residual but more artifacts. Fig.2 b) is chosen as the final solution based on the "L" curve: before artifacts become obvious yet with low residual. Positive susceptibility values were shown with intensity units of parts per million (ppm). SWI is a product of image intensity multiplied by the phase map [4]. SWI enhances the visibility of the hemorrhage with negative contrast, shown in Fig. 2 c) but is not quantitative. QSI perceives the hemorrhage with a positive contrast (iron creates positive susceptibility), shown in Fig. 2 b). Enlargement of hemorrhage (blooming) is shown in Fig. 3, where various susceptibility values of the hemorrhage correspond to different stages of the hemorrhage. The outer bright ring indicates hemorrhage converted into hemosiderin in Fig. 3 c).

A maximum intensity projection (MIP) of a slab with thickness of 26 mm clearly shows the distribution of the microbleeds and veins (Fig. 4 a)). With a narrow range of susceptibility (0 ~ 0.2 ppm), gray matter (white arrow, ~0.02 ppm) is perceived. Fig. 5 shows QSI of thalamus. More hemorrhages are visible but the parenchyma does not show excessive iron accumulation.

Phantom experiments with various concentrations of gadolinium (Gd) show a good estimation of susceptibility distribution. However, the susceptibility measurements are biased from the actual values by a factor of about 0.7. Accordingly, for QSI of the brain, susceptibility estimates are expected to contain a global scaling factor.

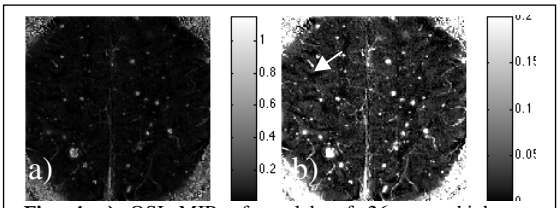

Fig. 4 a) QSI MIP of a slab of 26 mm thickness. Microbleeds and veins within the slab are recognized. **b)** QSI MIP with a smaller range of susceptibility values (0~0.2ppm). Estimated susceptibility is about 0.02 ppm for gray matter (white arrow).

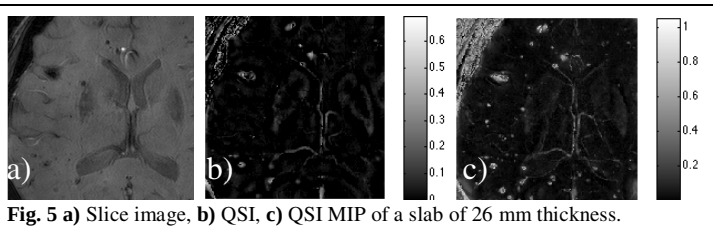

Fig. 5 a) Slice image, **b)** QSI, **c)** QSI MIP of a slab of 26 mm thickness.

CONCLUSIONS

Measurement of iron in brain can help understand brain functions and identify diseases. Methods attempting to quantitatively measure iron loading include MRI, T2 ($T2^*$) map, phase imaging, and SWI. Here we proposed an efficient approach based on homotopic L_0 norm minimization programming to solve the inverse problem from magnetic field measurement to susceptibility map. Results of brain exams have demonstrated the quantitative visibility of hemorrhage, veins and gray matter.

REFERENCES

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