Diffusion Weighted Imaging of Human Articular Cartilage using Fractional Calculus Model: Preliminary Study

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INTRODUCTION

Restricted diffusion doesn't follow the mono-exponential decay. Some reports have suggested using bi-exponential model. However, bi-exponential fitting is nontrivial and needs some experience to get fast and slow diffusion coefficients. In this paper, we generalize a spatial Laplacian in the Bloch-Torrey equation to incorporate a fractional order Brownian model of diffusivity. A new parameter β was derived from the new equation of Stejskal-Tanner gradient pulses. We fitted the signal attenuation obtained from the diffusion-weighted images of Sephadex gels and human articular cartilages using fractional order diffusion model. The results show that β can be used to reflect the structure of the tissue. The β value obtained from the osteoarthritis (OA) cartilage was larger than from the normal cartilage. Future development of this approach may be useful for detecting the early degeneration of OA.

THEORY

Assume that a fractional order generalization of the Bloch-Torrey equation for the transverse magnetization in the rotating frame can be written as:

$$\tau^{\alpha-1}\,{}_0^C D_t^{\alpha} M_{xy}(\mathbf{r},t) = \lambda M_{xy}(\mathbf{r},t) + D\mu^{2(\beta-1)}\nabla^{2\beta} M_{xy}(\mathbf{r},t)$$

where $\lambda = -\mathbf{i}\gamma(\mathbf{r}\cdot\mathbf{G}(t))$, ${}_0^C D_t^{\alpha}$ is the Caputo form of the Riemann-Liouville fractional order derivative in time [1], $\nabla^{2\beta} = \left(D_x^{2\beta} + D_y^{2\beta} + D_z^{2\beta}\right)$ is a sequential Riesz fractional order Laplacian operatior in space [1,2], and $\tau^{\alpha-1}$ and $\mu^{2(\beta-1)}$ are fractional order time and space needed to preserve units, ($0 < \alpha \le 1$, and $1/2 < \beta \le 1$) [3]. When $\alpha = 1$ and $\beta = 1$, the fractional differential operators become the usual integer order time and space partial derivatives and the equation becomes the classical Bloch-Torrey equation. For $\alpha = 1$ and an arbitrary β, the diffusion is assumed to follow fractional order dynamics in space. Given a π/2 spin echo pulse sequence with Stejskal-Tanner gradient pulses (G_z, Δ, δ), the result is:

$$M_{xy} = M_0 \exp\left[-D\mu^{2(\beta-1)}(\gamma G_z\delta)^{2\beta}\left(\Delta - (2\beta-1)/(2\beta+1)\delta\right)\right]$$

Also here, when $\beta = 1$, we can get the classical equation.

MATERIALS AND METHODS

Sephadex (G-25, G-50 or G-100) forms a dextran-water gel when the dry powder (20-50 μm, dia.) swells in water to form beads with many small interconnecting pores. The numerical value of the Sephadex refers to the approximate molecular exclusion size (in kilDaltons, kDa) of the pores.

Human tali were obtained within 24 hours of death of the donor through the Gift of Hope Organ and Tissue Donor Network (with Rush University IRB approval), and fixed in the 10% Formalin. The cartilage was cut into small pieces and was inserted in a 5 mm wide NMR tube filled with Formalin.

All diffusion experiments were conducted at 11.74 T (500 MHz for protons) using a Bruker DRX Avamce spectrometer. The diffusion measurements were carried out using a spin echo pulse sequence with Stejskal-Tanner diffusion gradients. For the DWI experiment in x direction of Sephadex, TR/TE = 1000/15 ms, slice thickness = 1.5 mm, matrix 128 x 128, FOV = 6 x 6 mm. and maximal b-value = 2000 s/mm^2. For the DWI experiments in x, y and z direction of the human articular cartilage, TR/TE = 3000/14 ms, slice thickness = 1mm, matrix 128 x 128, FOV = 6 x 6 mm. and maximal b-value = 3000 s/mm^2.

RESULTS

The figures and table show the β values of Sephadex (G-25, G-50 and G100) and β values of each zone (superficial, middle and deep zone) between normal and OA cartilage

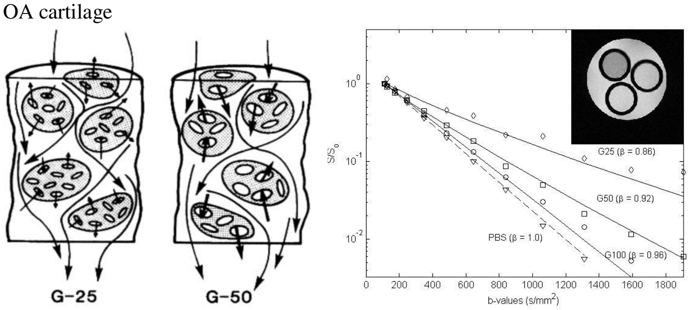

Fig. 1. Drawing and fitting curves of the Sephadex G-25, G-50 and G-100

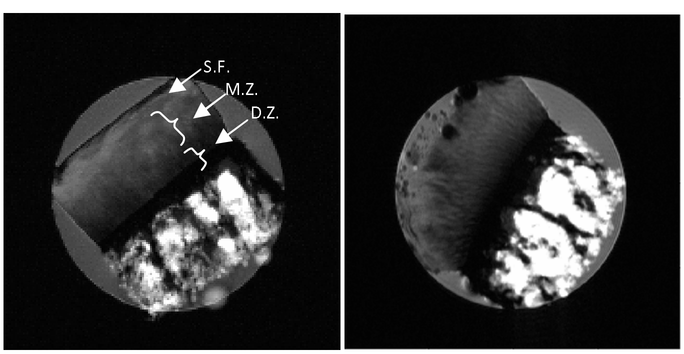

Fig. 2. Diffusion weighted image (DWI) of normal (left) and OA (right) cartilage. The superficial, middle and deep zones are marked as S.F., M.Z. and D.Z. shown in the figure

Table 1 Fractional diffusion coefficient (FDC) and β of each zone between normal cartilage and OA cartilage

	Zone	FDC(×10^{-3} mm^2/s)	β
Normal	Superficial	1.05±0.03	0.91±0.04
	Middle	0.96±0.01	0.92±0.01
	Deep	0.75±0.03	0.94±0.01
OA	Superficial	1.17±0.01	0.97±0.01
	Middle	1.12±0.08	0.98±0.01
	Deep	0.77±0.04	0.94±0.02
Increase percentage	Superficial	11.74%	5.91%
	Middle	16.46%	6.74%
	Deep	2.59%	0.92%

CONCLUSIONS

The β values of Sephadex gels confirm our hypothesis that the smaller the G-value of the Sephadex, the smaller the β value, because there are more restricted diffusion in the smaller pore size.

The collagen is the predominant extracellular matrix protein in cartilage. The collagen plays an important role in the restricted diffusion of cartilage DWI. OA usually starts with the collagen cleavage and proteoglycan loss from the superficial zone to the deep zone. The increasing of FDC value may imply the swell of cartilage in OA and the increasing of β value may imply the damage of collagen in OA. Our preliminary study shows that β can be looked as a marker of early degeneration of human cartilage.

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