The Equivalent Magnetization Current Method Applied to the Design of Gradient Coils for MRI

H. Sanchez[1], M. Poole[1], F. Liu[1], and S. Crozier[1]

[1]School of Information Technology & electrical Engineering, The University of Queensland, Brisbane, QLD, Australia

Synopsis: This paper presents an alternate approach for designing gradient coils that is independent of the shape of the current-carrying surface. The approach employs the equivalency between a uniformly magnetized volume and a surface current density. A linear variation of the magnetization in each boundary element was assumed which is equivalent to a uniform current density. A suite of electromagnetic properties can be parameterised in terms of a thin, piecewise, linearly-magnetised shell; the magnetic flux density, stored energy, power, torque, force and eddy-current induced magnetic flux density were all considered. A quadratic programming (QP) optimization algorithm was employed to find the magnetisation distribution that satisfies the various constraints of the electromagnetic design problem. Examples of coils are presented to demonstrate this method, including a novel gradient coil for breast imaging.

Method: Our model considers an isotropic, rigid, non-hysteretic, arbitrary volume of thickness *t*, bounded by the surface *S* ($S\in\Re^3$) and possessing a magnetization, **M**(**r'**), normal to *S*. A volume of linearly-varying magnetization **M**(**r'**) is equivalent to a uniform current density on its surface. If *t* is small and **M**(**r'**) is piecewise-linear throughout the thin volume it can be shown that

$$\mathbf{M}(\mathbf{r}')t=\psi(\mathbf{r}')\cdot\hat{\mathbf{n}}(\mathbf{r}'),\qquad \mathbf{r}'\in S,$$

where $\psi(\mathbf{r}')$ is the stream-function of the equivalent current density flowing on *S* and $\mathbf{n}(\mathbf{r}')$ is the normal vector to *S* at **r'**. The arbitrarily–shaped thin volume is discretized into N_E triangular elements with *N* nodes. We represent the stream-function, $\psi(\mathbf{r}')$, as a sum of unknown stream-function values, s_n, and basis-functions, $\psi_n(\mathbf{r}')$, which are of the form

$$\hat{\psi}_n(\mathbf{r}')=\left(1-\frac{(\mathbf{r}'-\mathbf{r}_{ni})\cdot\mathbf{d}_{ni}}{\left|\mathbf{d}_{ni}\right|^2}\right)\hat{u}(\mathbf{r}'),\qquad i=1,\ldots,N_n,\qquad \hat{u}(\mathbf{r}')=\begin{cases}1, & \mathbf{r}'\in\Delta_{ni}\\ 0, & \mathbf{r}'\notin\Delta_{ni}\end{cases}$$

where $\mathbf{d}_{ni}$ is the perpendicular distance vector from $\mathbf{r}_n$ to the far side of the triangle, Δ_{ni}. The magnetic flux density produced at a point **r** is then

$$\mathbf{B}(\mathbf{r})=-\frac{\mu_0}{4\pi}\sum_{n=1}^{N}s_n\sum_{i=1}^{N_n}\nabla_{\mathbf{r}}\int_{A'}\hat{\mathbf{n}}_{ni}(\mathbf{r}')\cdot\hat{\psi}_n(\mathbf{r}')\cdot\nabla_{\mathbf{r}'}\left(\frac{1}{|\mathbf{r}'-\mathbf{r}|}\right)dA'$$

Using the Equivalent Magnetizing Current (EMC) concept, equations for the stored magnetic energy, power dissipation, force, torque and eddy-current (EC) effects can be derived that are of the same form as those presented by Peeren [2]. However, the calculation of these properties can be speeded up by assuming that the magnetisation is piecewise uniform in the thin volume rather than piecewise linear. When considering the EC induced in conducting surfaces it is assumed that the current in the gradient coil is switched on "instantaneously" and that the stream-function of the EC, s_{nEC}, are linked to s_n by

$$s_{nEC}=-\mathbf{M}_{cc}^{-1}\mathbf{M}_{cs}s_n$$

where $\mathbf{M}_{cc}$ and $\mathbf{M}_{cs}$ are matrices that describe the self-inductance of the cryostat and the mutual inductance between coil and cryostat respectively.

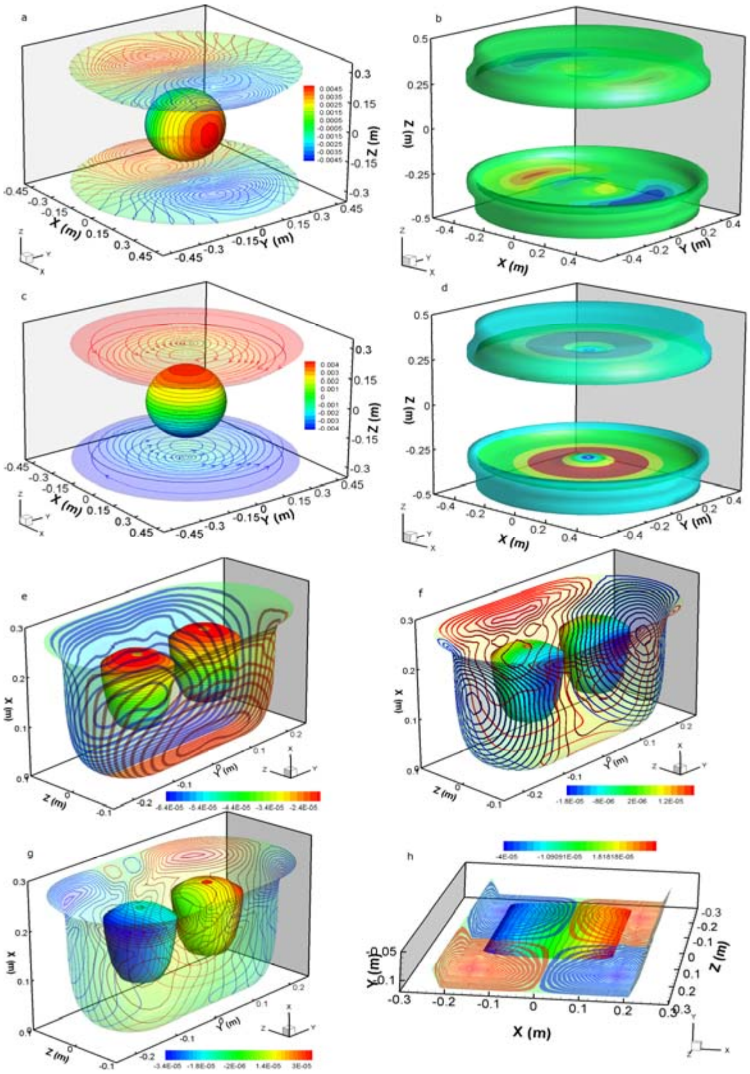

Figure 1. Bi-planar (a) transverse and(c) longitudinal gradient coils. Colour and arrow indicate the current direction. The instantaneous Eddy current induced in the conducting surface produced by the transverse (b) and longitudinal coils (d). Current pattern of the (e) x-, (f) z- and (g) y-gradient coils for breast imaging. (h) An insertable, planar x-gradient coil design. The colour scale represents the values of the axial magnetic flux density component in Tesla.

The magnetic flux density, $B_z(\mathbf{r})$, force, **F**, torque, **T**, and EC-induced magnetic flux density, $B_{zEC}(\mathbf{r})$, are linear with respect to s_n and the power dissipation, *P*, and stored energy, *W*, are quadratic. The optimization problem was therefore stated as a quadratic programming (QP) problem, subject to linear constraints:

$$\min(P,W),\qquad \text{s.t.}\qquad \frac{\pm\partial B_z(\mathbf{r})}{\partial\mathbf{r}}\le\pm G_0(1-\varepsilon_1),\ \pm B_{zEC}(\mathbf{r})\le\varepsilon_2,\ \ \mathbf{F}=0,\ \ \mathbf{T}=0$$

We employed the *quadprog* function provided in the MATLAB® optimization toolbox. The software FastHenry [3] was used to validate the impedance calculation and GMSH [4] was employed to generate the surface meshes.

To demonstrate the EMC method for designing gradient coils with complex geometry, three different coil geometries have been considered: **a)** a bi-planar coil for open magnets where the primary and secondary coil have been joined through a conical surface, **b)** a new coil for breast imaging with 2 DSVs and **c)** an insertable uniplanar geometry [5].

Results and Discussions: Figs. 1, (a,c) show the wire-paths for transverse and longitudinal biplanar gradient coils for an open system. The turns may to flow from the primary to the secondary surfaces via the conical connecting surface. A figure-of-merit (FoM), $\eta^2/L = 0.14\cdot10^{-4}$ ($T^2m^{-2}A^{-2}H^{-1}$) was obtained. The inclusion and control of the EC as linear constraints produces smooth current pattern solutions even when a small distance (1-4 cm) separates the shielding coil to the conducting surface. Figs. 1, (b,d) show the EC induced (colour in arbitrary units) in realistic conducting pole surfaces for the 2 coils in (a,c). Using 3mm square wire the coils (a and c) produce resistances of 223 mΩ and 102 mΩ respectively. Figs. 1 (e,f,g) show wire-paths for the new double DSV gradient coils for breast imaging. A highly-linear magnetic flux density is produced (<5%) with a FoM of $11.1\cdot10^{-4}$ ($T^2m^{-2}A^{-2}H^{-1}$), $10.5\cdot10^{-4}$($T^2m^{-2}A^{-2}H^{-1}$) and $9.0\cdot10^{-4}$($T^2m^{-2}A^{-2}H^{-1}$) for the *x*-,-*z* and *y*-gradient coils respectively. The use of two target DSVs instead of one large DSV leads to higher performance. The insertable *x*-gradient coil showed in Fig. 1 (h) possesses a FoM of $5.5\cdot10^{-4}$ ($T^2m^{-2}A^{-2}H^{-1}$) and a resistance of 34.4 mΩ using copper strips with width varying from 2.1 mm to 14 mm accommodating a track gap of 1.25 mm and constant 3.2 mm thickness.

Conclusions: An alternative approach for designing gradient coils that are independent of the shape of the current-carrying surface has been presented. This method produces high performance coils and opens the possibility of simultaneously designing hybrid coil-iron structures in which magnetic material and current-carrying wires that may occupy different domains. A novel coil structure and new coil patterns for localised breast imaging have been presented as well bi-planar gradients for open systems and an insertable local gradient coil.

References: [1] Pissanetzky S. 1992 Meas. Sci. Technol; 3: 667-673. [2] Peeren GN. 2003 Journal of Computational Physics;191:305–321. [3] M. Kamon, M.Tsuk, J. White. 1994 IEEE Trans MTT;42:1750–1758. [4] C. Geuzaine, J.-F. Remacle. http://www.geuz.org/gmsh/; 2008.[5] Forbes LK and Crozier S 2004 IEEE Trans Magn;40:1929-1938.

Acknowledgements: Financial support from the Australian Research Council is gratefully acknowledged.