

# Automatic Design of Radial Trajectories for Parallel MRI and Anisotropic Fields-of-View

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**Introduction:** In recent years, non-Cartesian  $k$ -space trajectories such as radials and spirals have gained increased attention owing to mild appearance of artifacts from flow, chemical shift and undersampling, increased efficiency and intrinsic self-navigating properties. Remaining challenges of imaging with non-Cartesian trajectories include the lack of practical methods to optimize their performance under object- and system-specific imaging constraints. The examples include optimization to anisotropic Fields-of-View (FOV) arising from slab-selective excitation and anatomy, and design maximizing parallel imaging performance for a given coil array. Heuristic optimization of radials to anisotropic FOV [1] demonstrated potential of such designs to decrease significantly data sampling overhead for many existing non-Cartesian trajectories. While Cartesian scans may be readily tuned to perform efficiently with both anisotropic FOV and a given coil array in parallel imaging mode [2], no general automatic method exists to perform such optimization for non-Cartesian trajectories.

We propose a new approach to the design of non-Cartesian trajectories based on the analysis of properties of matrix inversion underlying non-Cartesian data reconstruction. We reduce the complexity of matrix inversion by approximating the matrix inverse with a local  $k$ -space based reconstruction such as PARS/GRAPPA [3, 4]. The design of optimized trajectories is guided by probing the ability of the inverse to reliably solve the inverse problem of restoring image content from samples on a  $k$ -space trajectory. We apply the developed theory to create a fast method that optimizes radials for arbitrary FOV and parallel MRI with a given coil array.

**Theory:** The MRI signal encoding in the matrix form is given by  $\mathbf{E}\mathbf{f} = \mathbf{s}$ , where  $\mathbf{f}$  is the object,  $\mathbf{s}$  is  $k$ -space signal from coil receivers,  $\mathbf{E}$  is the encoding matrix with elements,  $[\mathbf{E}]_{(m,\gamma),\rho} = e^{ik_{m,\gamma}\rho} c_{\gamma}(r_{\rho})$ ,  $c_{\gamma}$ ,  $\gamma = 1, \dots, N_c$  are coil sensitivities. The ability to robustly invert the encoding matrix and restore the underlying image depends on the condition number of  $\mathbf{E}$ , which reflects how sensitive reconstruction is to noise errors in data samples, and which directly affects  $g$ -factor of parallel MRI reconstruction.

One way to optimize the  $g$ -factor is to minimize the trace  $tr(\mathbf{E}^H\mathbf{E})^{-1}$  [2]. However, direct optimization is infeasible for non-Cartesian trajectories because of the size of the matrix  $\mathbf{E}$ . Instead, we start with an observation that  $tr(\mathbf{E}^H\mathbf{E})^{-1} = \|\mathbf{E}^{\dagger}\|_2$ , where

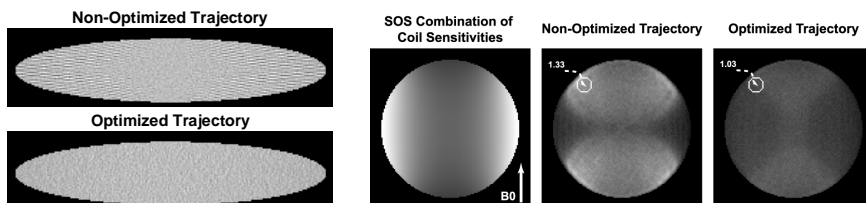
$\mathbf{E}^{\dagger}$  is the Moore-Penrose pseudoinverse. Next, we note that an equivalent pseudoinverse  $\mathbf{P}$  may be efficiently built in  $k$ -space using approximate local techniques such as GRAPPA or PARS [6]. For optimized sparse approximation,  $\|\mathbf{E}^{\dagger}\|_2 \approx \|\mathbf{P}\|_2$  [5].

Hence, minimization of  $g$ -factor may be done minimizing norm of the rows of  $\mathbf{P}$ , which are comprised of GRAPPA/PARS coefficients. We propose to use two error metrics associated with such approximation: residual error  $\varepsilon$  and noise factor  $\eta$  (norm of GRAPPA/PARS coefficients). Our optimization of non-Cartesian trajectories using GRAPPA/PARS coefficients consists of two steps: 1) Initial trajectory design to minimize local residual errors  $\varepsilon$  of GRAPPA/PARS fits. 2) Final trajectory adjustment to minimize local noise factors  $\eta$ .

**Methods and Results: Optimization of radial trajectories:** For radial trajectories, the free design parameter is the angular distribution of radials. In our previous work [6], we have shown that PARS/GRAPPA coefficients are slowly varying functions in polar coordinates. Hence, both local approximation error  $\varepsilon$  and noise factor  $\eta$  also vary smoothly and their distribution in polar coordinates could be approximated by values at few reference points. The maps of  $\varepsilon$  and  $\eta$  reflect the dependence of inversion quality on local sampling density, which is subsequently used to choose proper angular spacing. The optimization procedure is shown in Fig. 1. **Simulations:** Figure 2 demonstrates the effect of the proposed trajectory optimization on image quality in the case of anisotropic FOV. Streaking artifacts visible with non-optimized trajectory are removed when the same number of optimally distributed projections is used. To demonstrate the effect of trajectory optimization on the  $g$ -factor, we simulated the  $x$ - $z$  plane of coil sensitivities of an 8 channel coil array. Readjusting the projection angles using proposed optimization allowed for a significant reduction of the peak  $g$ -factor (Fig. 3). **Human data:** The abdominal data from healthy volunteer were collected on a GE 1.5T SIGNA EXCITE (GE Healthcare, Milwaukee, WI) scanner using a 2D bSSFP radial sequence (flip angle 15, 4096 projections, 256 points/projection). Non-optimized and optimized acquisitions were simulated from the full datasets, and reconstructed using accelerated PARS approach [6]. Streaking artifacts are visibly reduced when applying parallel MRI reconstruction with optimized trajectory.

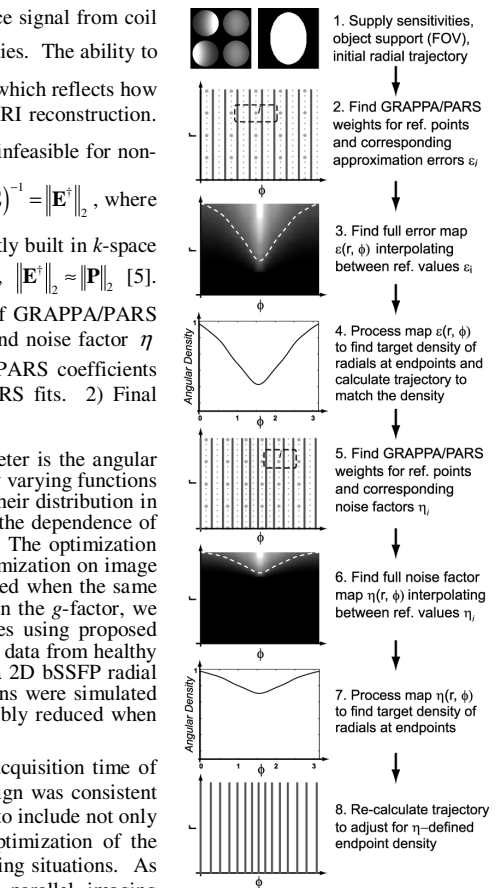
**Discussion:** We have developed an automatic procedure for design of radial trajectories that minimizes acquisition time of non-symmetric volumes and improves parallel MRI performance. For anisotropic FOV, the trajectory design was consistent with a previously proposed approach [1]. Our technique is a general trajectory design tool, which is flexible to include not only FOV information but also any prior image such as scout/localizer scans. Simultaneously, it allows optimization of the trajectory for a given coil array to reduce parallel MRI  $g$ -factor. The latter is of importance in many imaging situations. As anatomical restrictions on placement of coil elements often precludes isotropic sensitivity encoding, parallel imaging performance in certain directions may be jeopardized (Fig. 3). Hence, optimization as proposed may be especially beneficial for true 3D and certain oblique planes of 2D trajectories to compensate for the “dead zones” of sensitivity encoding. The design ideas may be readily extended to other non-Cartesian trajectories such as spirals.

**Acknowledgements:** We would like to acknowledge NCI R01CA116380 and GE Healthcare. **References:** [1] Larson PEZ, et al. IEEE TMI, 2008, 27(1):47. [2] Jacob M, et al. PI Workshop, Zurich, 2004. [3] Yeh EN, et al. MRM, 2005, 53:1383. [4] Griswold MA, et al. ISMRM 2003, 2349. [5] Samsonov AA, MRM, 2008, 59:156. [6] Samsonov AA, et al. MRM, 2006, 55(2):431.

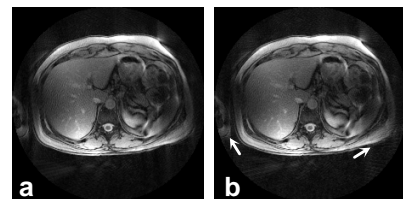


**Figure 2:** Results of automatic optimization of radials for an ellipse FOV (181 projections in each case, full sampling is 400 projections).

**Figure 3:** Simulated  $g$ -factors for uniform 2D radial and optimized trajectories (reduction factor 2.5, simulated 8 coil-concentric array, iterative SENSE). The trajectory optimization reduced peak  $g$ -factor value by 1.29 times.



**Figure 1.** Design of radial trajectory for prescribed FOV and/or parallel MRI.



**Figure 4:** Optimized parallel MRI with optimized radials (4 coil receivers, 256x256, reconstruction from 81 to 324 projections). **a:** Optimized trajectory. **b:** Nonoptimized trajectory. Note elevated streaks in **b** (white arrows).