Noise Behavior of Cartesian PatLoc Reconstruction

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Introduction: Image quality strongly depends on the noise behavior of the underlying imaging method. In particular it depends on the error propagation properties of the applied reconstruction algorithm. The purpose of this work is to perform a noise analysis in the context of PatLoc imaging [1]. In this imaging modality the gradients are replaced by coils, which generate nonlinear and non-bijective encoding fields. A practical example of such fields is shown in Fig. 1,3 and Ref. [2]. PatLoc imaging allows one to overcome current limitations of peripheral nerve stimulation and to design encoding fields, which fit to the non-Cartesian geometry of the anatomy. Non-bijective encoding leads to ambiguities, which can be resolved by means of a suitable parallel imaging reconstruction method. Here an efficient reconstruction algorithm [3] for Cartesian sampling strategies is considered. The simple structure of this algorithm enables one to derive a clear mathematical description from which interesting conclusions can be drawn. This algorithm is a linear reconstruction method belonging to the broader class of image-based parallel imaging reconstruction methods. A common feature of such methods is that the SNR varies from point to point. For every new parallel imaging method it is therefore crucial to perform an exact noise analysis.

Theory: For a linear reconstruction method described by matrix F, a noise analysis can be performed by means of correlation matrices (see ref. [4] and Fig. 4). Consider white noise in the receiver coils. Then it is possible to describe the noise in the receiver coils by a noise correlation matrix Ψ. A block-diagonal signal correlation matrix $\tilde{\Psi} = \Psi \otimes Id_k$ can be defined, where k is the number of acquired k-space points. The image noise matrix X can be calculated as $X = F\tilde{\Psi}F^{\dagger}$ (see [4]). The diagonal matrix elements of X then represent the variance of the noise in the reconstructed image. Then, the SNR in each voxel ρ can be calculated directly as:

(1) $$SNR_{\rho}^{PatLoc} = \frac{SNR_{\rho}^{full}}{\sqrt{R d_{\rho}} g_{\rho}}$$

Figure 1: Prototype of a PatLoc gradient coil.

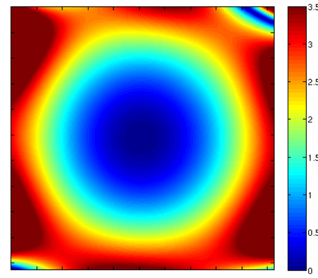

Figure 2: Intensity correction d_{ρ} induced by the multipolar encoding fields of second order (Fig. 3a,c).

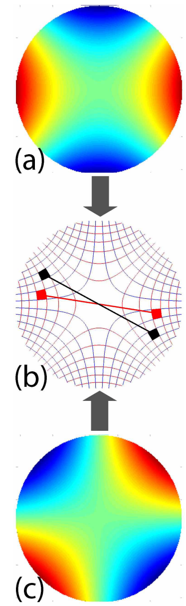

Figure 3: (a) and (c): Multipolar PatLoc-encoding fields of second order. (b): Achievable resolution. The colored pixels are examples of pixel groups.

This equation reminds one of the g-factor defined in Eq. 24 in [4]. SNR_{ρ}^{full} is the optimized SNR in voxel ρ, when linear gradient fields are used for encoding. R describes the intrinsic acceleration, an inherent property of PatLoc imaging. g_{ρ} is the famous g-factor. Similar to the conventional case it is constructed by groups of identically encoded pixels. However, in the case of the mulitpolar fields as shown in Fig. 3a,c, these groups are not formed by equidistant pixels, but rather by pixels, which lie on opposite sides (Fig. 3b). The correction factor d_{ρ} describes an intensity correction, which becomes necessary because of the nonlinear nature of the encoding fields. It is given by the absolute value of the determinant of the Jacobian of the vector field, whose components are formed by the encoding fields. The multipolar fields make an intensity correction as depicted in Fig. 2 necessary.

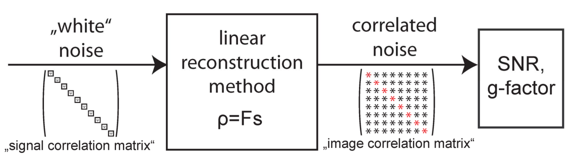

Figure 4: Linear PatLoc-reconstruction methods allow the analysis of their noise behavior by the use of correlation matrices.

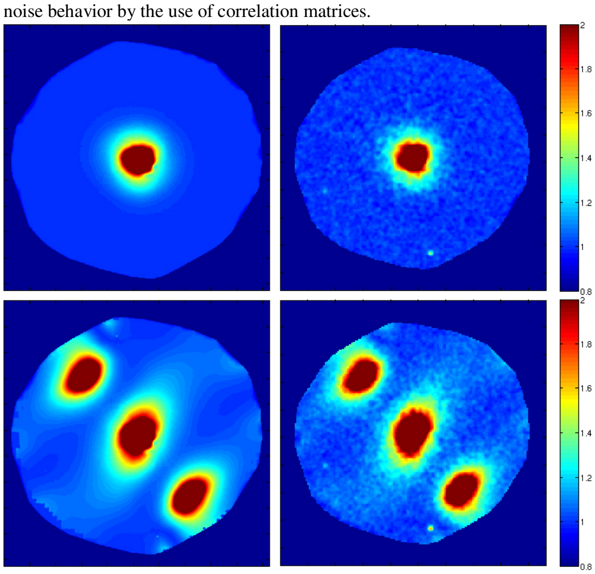

Figure 5: Left: Calculated g-factor maps according to Eq. (1). Right: Respective simulation results. In the upper images eight receiver coils have been considered, in the bottom images only two channels lying on opposite sides.

Methods: To validate Eq. 1 by means of numerical simulation, the simulations were based on measured sensitivity maps of several receiver coils and measured field maps of the encoding fields. The used encoding fields were similar to the fields generated by shim coils of second order (Fig. 3a,c). With this data g-factor g_{ρ} and intensity correction d_{ρ} were calculated according to Eq. 1 for each pixel. Simulations were performed by adding Gaussian white noise to the signal. One image was reconstructed with the PatLoc-reconstruction algorithm, another one with conventional SNR-optimized multi-coil reconstruction [5]. Eq. 1 was solved for g_{ρ} and with the pre-calculated correction term d_{ρ} we computed the g-factor based on simulation data. This procedure was repeated 100 times to get a lower error on g_{ρ}.

Results and Discussion: The theoretical predictions made by Eq. 1 and simulation results for g_{ρ} are compared in Fig. 5. Good agreement between calculation and simulation is apparent. So, Eq. 1 could successfully be validated.

This equation resembles the formula obtained for Cartesian SENSE [4]. In fact, it generalizes the previously found result because in PatLoc imaging also nonlinear fields are considered. Such fields cause non-uniform resolution in the reconstructed images. Fig. 3b shows that the multipolar fields generate a grid with increasing resolution towards the periphery. The intensity correction is required in PatLoc imaging due to the non-uniformity of resolution and it is proportional to it. This property can be seen by comparing Fig. 2 with Fig. 3b. As expected, the SNR scales linearly with voxel size.

Another interesting aspect is that PatLoc imaging using multipolar fields and parallel imaging fit very well together. Fig. 3b indicates that magnetization on opposite sides is identically encoded. When receiver coils are placed around the object the sensitive regions focus very well on one side of the object, whereas the opposite side is almost not visible for the same coil. This means that the g-factor is closely to unity at least when more than two receiver coils are used. Fig. 5 shows that only in the center of the imaged region, where imaging is not possible, the g-factor is not near to unity.

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References: [1] Hennig et al., MAGMA 21(1-2):5–14 (2008); [2] Welz et al., Proc. ISMRM 2008, #1163, Toronto (2008); [3] Schultz et al., Proc. ISMRM, #786, Toronto (2008); [4] Pruessmann et al., MRM 42, 952–962 (1999); [5] Roemer et al., MRM 16, 192–225 (1990).