Oscillating Radial Trajectories for Reduced Undersampling Artifacts

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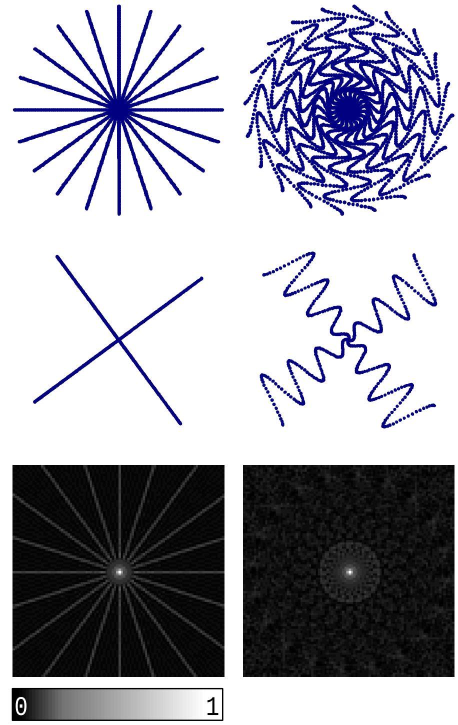


Fig 1: Traditional radial sampling (left) and oscillating radial sampling (right). Top row shows *k*-space sampling for ten radial arms. For clarity, second row displays only two of the ten radial arms shown in the first row. Third row shows PSF corresponding to the first row.

Introduction: In this work, we propose an oscillating radial sampling of *k*-space which shows a significant improvement in the reconstruction quality over the traditional radial sampling based reconstruction. The incoherency in spatial aliasing artifacts is selected as a criterion for optimizing the *k*-space trajectory. Adding oscillations reduces the coherency in the *k*-space trajectory and hence minimizes the aliasing artifacts. In contrast to the other methods for generating randomized trajectories [1], the proposed *k*-space trajectories are smooth and hence easy to implement on a conventional MRI gradient coil system. We present a systematic way of generating oscillating radial trajectories and show an improvement over the traditional radial sampling using simulations.

Theory: Non-Cartesian trajectories (e.g. radial and spiral) have received an increased attention in various applications including cardiac imaging, functional brain imaging, contrast-enhanced MR angiography, and hyperpolarized gas imaging. One major drawback of the non-Cartesian sampling is the enhanced aliasing artifacts which can partially be attributed to the coherence in the *k*-space sampling pattern, resulting in the point-spread-function (PSF) intensity leaking into the sidelobes [2]. A straight forward approach to reduce the aliasing artifacts is to sharpen the PSF.

Method: First we parameterize the radial trajectory by using Eq. 1.

$$\begin{aligned} kx(\tau,\theta) &= \tau\cos\theta + a\,\mathrm{sign}(\tau)\left|\tau\right|^{w\sin\theta}\cos(2\pi f\tau) \\ ky(\tau,\theta) &= \tau\sin\theta - a\,\mathrm{sign}(\tau)\left|\tau\right|^{w\cos\theta}\cos(2\pi f\tau) \end{aligned} \qquad (1)$$

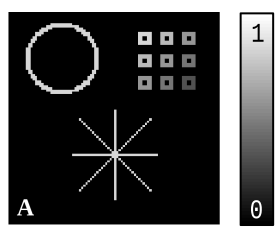
In Eq. 1, kx and ky define x and y coordinates of the trajectory, $\tau \in (-0.5, 0.5)$ defines pseudo time axis, $a \in (0,1)$ represents the amplitude of sinusoidal oscillations, $f \in (0,10)$ defines the frequency of oscillations, θ describes the orientation of a radial arm, and $w \in (0.5,1)$ controls the increase in oscillation magnitude as a function of τ. For $w = 1$, the oscillation amplitude increases linearly with τ. We have selected incoherency in the *k*-space sampling as a criterion to optimize the trajectory. An effective method of estimating the incoherency is to measure the intensity E of the PSF that has leaked into the sidelobes. We computed PSF using nonuniform fast Fourier transform [3] of *k*-space. To compute E, we adopted Eq. 2.

$$E = \left\| \mathrm{PSF} \times (x^2 + y^2)^{1/4} \right\|^2 \qquad (2)$$

In Eq. 2, "$\times$" describes point-by-point multiplication. Now the problem of finding an optimized oscillating radial trajectory $K(\vec{\psi})$, for a given number of radial arms, translates to finding (a, w, f) for which E is minimized, as given by Eq. 3.

$$\vec{\psi} = \underset{a,w,f}{\arg\min}\, E \qquad (3)$$

In this model, since there are only 3 parameters to be adjusted each over a narrow range, we used linear search to find the optimum values of $a, w,$ and f. For a trajectory with 5 radial arms, the optimum values of $a, w,$ and f were found to be 0.39, 0.6, and 4 respectively, while for a trajectory with 10 radial arms, the optimum values of $a, w,$ and f were calculated to be 0.2, 0.6, and 7 respectively. A more comprehensive model, for example the one where each arm of radial acquisition has separate tuning parameters, can also be implemented albeit at a higher computational cost. For reconstruction, we used an iterative method [4] for TV-regularization, with significant acceleration achieved by exploiting the Toeplitz-block-Toeplitz nature of the problem [5].



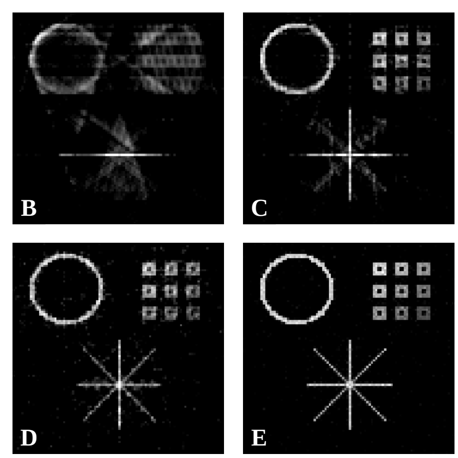


Fig 2: Reconstruction results. Numerical phantom (A). reconstruction from traditional radial sampling with five arms (B), ten arms (C), with the proposed oscillating radial sampling with five arms (D), ten arms (E).

Results: A 96×96 numerical phantom, shown in Fig. 2A, was used for the simulation studies. Figures 2B and 2C show the reconstruction from the traditional radial trajectories with 5 and 10 arms respectively, while Figs. 2D and 2E show the reconstruction from the proposed oscillating radial trajectories with 5 and 10 arms respectively. For a 5 arm optimized trajectory, the value of E was reduced by 57% while the mean-square-error (MSE) of reconstruction was reduced by 58% in comparison to the traditional radial sampling. Likewise, for a 10 arm optimized trajectory, the value of E was reduced by 56% while the reconstruction MSE was reduced by 81% in comparison to the traditional radial sampling.

Conclusion: We have presented a systematic way of adding oscillations to the traditional radial sampling, resulting in a sharper PSF with reduced aliasing artifacts. The *k*-space trajectories are smooth and can be implemented with a conventional MRI gradient coil systems.

Reference: *[1] A. Bilgin, T. P. Trouard, A. F. Gmitro, and M. I. Altbach, ISMRM, Toronto, 2008. [2] M. L. Lauzon and B. K. Rutt, MRM, 36, 1996. [3] S. Kunis and D. Potts, JCAM, 161, 2003. [4] M.A.T. Figueiredo, R.D. Nowak, S.J. Wright, IEEE, 2007. [5] A.H. Delaney and Y. Bresler IEEE 1995.*