Improving SNR by Generalizing Noise Matching for Array Coils

C. Findeklee[1]
[1]Philips Research Europe, Hamburg, Germany

Introduction:
The noise from the preamplifiers (LNAs: low noise amplifiers) together with the thermal noise due to losses in patient and coil elements, determines the final signal-to-noise ratio (SNR) of the receive chain. A single LNA performs best, when the matched coil presents the so-called optimum noise impedance to the preamplifier. However, an array coil presents an impedance matrix to the LNAs which typically has non-diagonal elements due to element coupling. As shown in many publications, e.g. [Rey00], this leads to noise coupling and reduces the final SNR, if the noise matching is not corrected for the coupling. This paper shows how the noise matching can be generalized for the case of array coils. This allows the coil designer to achieve true optimum SNR for a selected imaging region.

Theory:
Fig. 1 shows an equivalent circuit of a noisy single channel receive chain. The coil is represented by the impedance Z_c, the noise voltage $\underline{U}_c$ and the signal voltage $\underline{U}_s$.
The noise figure F of an LNA can be defined as the ratio of SNR of the input signal in absence of LNA-noise to the SNR of the output signal including LNA-noise – with the thermal coil noise $\overline{\underline{U}_c \underline{U}_c^{\mathrm{H}}} = \sim\left(Z_c + Z_c^{\mathrm{H}}\right)$. (Whereby Z^{H} denotes the complex conjugate of Z and the line over expressions denotes the covariances of the noisy sources.) The noise figure is a function of the coil impedance Z_c but is not a function of the preamplifier input impedance Z_{p}. The optimum coil impedance $Z_{\mathrm{c,opt}}$ for the lowest possible noise figure F_{opt} can be derived as:

$$Z_{\mathrm{c,opt}} =: R_{\mathrm{c,opt}} + \mathrm{j}X_{\mathrm{c,opt}} \quad , \quad \left|Z_{\mathrm{c,opt}}\right| = \sqrt{\frac{\overline{\underline{U}_{\mathrm{p}}\underline{U}_{\mathrm{p}}^{\mathrm{H}}}}{\overline{\underline{I}_{\mathrm{p}}\underline{I}_{\mathrm{p}}^{\mathrm{H}}}}} \quad , \quad X_{\mathrm{c,opt}} = \frac{\mathrm{imag}\,\overline{\underline{I}_{\mathrm{p}}\underline{U}_{\mathrm{p}}^{\mathrm{H}}}}{\overline{\underline{I}_{\mathrm{p}}\underline{I}_{\mathrm{p}}^{\mathrm{H}}}}$$

If the coil impedance $Z_{\mathrm{c}} =: R_{\mathrm{c}} + \mathrm{j}X_{\mathrm{c}}$ deviates from this optimum, the noise figure increases [Eng95]

$$F = F_{\mathrm{opt}} + \frac{G_{\underline{I}_{\mathrm{p}}}}{R_{\mathrm{c}}}\left|Z_{\mathrm{c}} - Z_{\mathrm{c,opt}}\right|^2$$

($G_{\underline{I}_{\mathrm{p}}}$ denotes the thermal noisy conductance that generates the same noise as $\underline{I}_{\mathrm{p}}$) and the SNR at the LNA output can be expressed by

$$SNR_{\mathrm{out}} \sim \underline{U}_s^{\mathrm{H}}\left(Z_{\mathrm{c}}F_{\mathrm{opt}} + F_{\mathrm{opt}}Z_{\mathrm{c}}^{\mathrm{H}} + 2\left(Z_{\mathrm{c}} - Z_{\mathrm{c,opt}}\right)G_{\underline{I}_{\mathrm{p}}}\left(Z_{\mathrm{c}} - Z_{\mathrm{c,opt}}\right)^{\mathrm{H}}\right)^{-1}\underline{U}_s.$$

Fig. 2 shows a two-element array coil attached to two preamplifiers with $Z_{\mathrm{p}} = \infty$ (w.l.o.g.). The noise current source $\underline{I}_{\mathrm{p},1}$ contributes with $-Z_{\mathrm{c};2,1} \cdot \underline{I}_{\mathrm{p},1}$ in port 2 and vice versa (noise coupling).
It can be shown that for the case of array coils the just derived SNR expression can be interpreted as a matrix equation, if the optimized combination, presented by Appelbaum [App86], Roemer [Roe90] and Pruessmann [Prue99], is used:

$$SNR_{\mathrm{out}} \sim \vec{\underline{\boldsymbol{U}}}_{\mathrm{s}}^{\mathrm{H}}\left(\boldsymbol{Z}_{\mathrm{c}}\left[F_{\mathrm{opt}}\right] + \left[F_{\mathrm{opt}}\right]\boldsymbol{Z}_{\mathrm{c}}^{\mathrm{H}} + \left(\boldsymbol{Z}_{\mathrm{c}} - \left[Z_{\mathrm{c,opt}}\right]\right)\left[2G_{\underline{I}_{\mathrm{p}}}\right]\left(\boldsymbol{Z}_{\mathrm{c}} - \left[Z_{\mathrm{c,opt}}\right]\right)^{\mathrm{H}}\right)^{-1}\vec{\underline{\boldsymbol{U}}}_{\mathrm{s}}$$

In this equation, the squared brackets have to be evaluated channel-wise forming diagonal matrices, and the parameters $Z_{\mathrm{c,opt}}$ and $G_{\underline{I}_{\mathrm{p}}}$ have to be interpreted as parameters of the single LNAs, including the matching circuits attached to the same. ($\boldsymbol{Z}^{\mathrm{H}}$ now denotes the transpose complex conjugate of $\boldsymbol{Z}$.)
Using network theory, a similar expression can derived for a weighted combination of power $\tilde{P}$. In this new function, the signal power received in each individual LNA impedance $Z_{\mathrm{p}} = R_{\mathrm{p}} + \mathrm{j}X_{\mathrm{p}}$ is divided by a corresponding real scalar Φ:

$$\tilde{P} \sim \vec{\underline{\boldsymbol{U}}}_{\mathrm{s}}^{\mathrm{H}}\left(\boldsymbol{Z}_c[\Phi] + [\Phi]\boldsymbol{Z}_c^{\mathrm{H}} + \left(\boldsymbol{Z}_{\mathrm{c}} - \left[Z_{\mathrm{p}}^{\mathrm{H}}\right]\right)\left[\frac{\Phi}{2R_{\mathrm{p}}}\right]\left(\boldsymbol{Z}_{\mathrm{c}} - \left[Z_{\mathrm{p}}^{\mathrm{H}}\right]\right)^{\mathrm{H}}\right)^{-1}\vec{\underline{\boldsymbol{U}}}_{\mathrm{s}}$$

It can be shown that for $[\Phi] := \left[F_{\mathrm{opt}}\right]$, both of the above functions have the same optimum impedance $\left[Z_{\mathrm{c,opt}}\right] = \left[Z_{\mathrm{p}}^{\mathrm{H}}\right]$, if $\boldsymbol{Z}_{\mathrm{c}}\left[F_{\mathrm{opt}}\right] + \left[F_{\mathrm{opt}}\right]\boldsymbol{Z}_{\mathrm{c}}^{\mathrm{H}}$ is positive-semidefinite. This means, that the optimum matching condition in many realistic cases can be found by modifying the LNA inputs such that they present the complex conjugate of their own single channel optimum impedance, and provide a matching such that the weighted power sum – each channel power divided by the individual (adapted) gain and noise figure – at the LNA outputs is maximized. Afterwards, the LNA input modification is removed.
If all LNAs have the same noise figure and the coil is reciprocal, this method of matching is equivalent to the method of using the active antenna impedance as proposed in [Maas07].

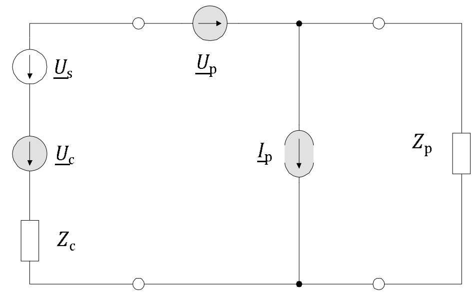

Fig. 1: Noisy single channel chain

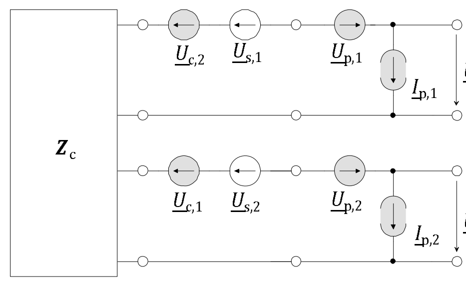

Fig. 2: Noisy chain for two elements

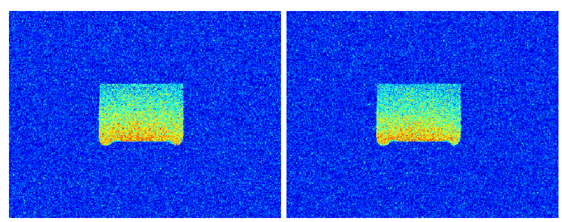

Fig. 3: Single element images using standard noise matching

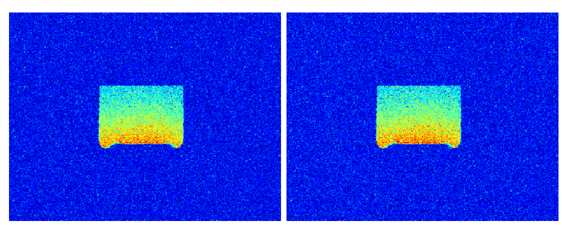

Fig. 4: Single element images using optimized matching

Methods/Conclusion:
The proposed noise matching method was used for matching two circular loops of 10cm diameter with a center distance of 11.25cm for optimized SNR in the central plane. Compared to a standard method with an improved preamplifier decoupling (Fig. 3), the SNR was improved by more than 6% (Fig. 4, new method, without preamplifier decoupling). Simulations show that for multi element coils with decoupled nearest neighbours, but residual mutual couplings from more distant coil elements, an even greater improvement can be expected.

References:
[App76] S.P. Appelbaum, IEEE AP 24, pp. 585-598, 1976
[Eng95] J. Engberg et. al: ISBN 0-471-94825-X, 1995
[Maas07] R. Maaskant et al.: AP Society Intern. Symposium 2007 IEEE, 5889-5892
[Prue99] K.P. Prüssmann et. al: MRM 42, pp. 952-962, 1999
[Rey00] A. Reykowski et. al: ISMRM 2000, p. 1402
[Roe90] P.B. Roemer et. al: MRM 16, pp. 192-225, 1990