L1-norm regularization of coil sensitivities in non-linear parallel imaging reconstruction

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Introduction: In parallel MRI, accurate coil sensitivity estimates are required to reconstruct aliasing free images [1, 2]. A promising alternative to the common estimation methods based on a low resolution pre-scan or on a fully sampled central *k*-space area is to perform a joint reconstruction of image and coil sensitivities. Existing approaches exploit the a priori assumption that coil sensitivities are smooth functions to regularize the non-linear reconstruction problem either by using a polynomial model for the sensitivities, as in JSENSE [3], or by penalizing their Sobolev norm, as in the non-linear inverse algorithm of [4]. In this work, a method inspired by Compressed Sensing that constrains the coil sensitivities to be sparse in a certain transform domain is presented and evaluated with in vivo data.

Theory: The acquisition of parallel MR data d can be modelled as the pixel-wise multiplication M(S,I) of an image I and coil sensitivities S, followed by an undersampled Fourier transform F_u. Under the assumption of Gaussian noise, image reconstruction can be performed by minimizing the cost function $\|d-F_uM(S,I)\|_2$. In a non-linear joint reconstruction framework, the minimization is performed with respect to both the image and the coil sensitivities. The number of parameters to be estimated is hence much higher than the number of data samples, so that the resulting inverse problem is not well-posed. According to the theory of Compressed Sensing [5], highly undersampled signals can be accurately reconstructed by l_1 norm minimization in a domain where they are sufficiently sparse. We propose to adapt this approach to the estimation of coil sensitivities by penalizing their l_1 norm in a domain Ψ_S, which yields the following minimization problem:

$$\min_{(I,S)} \|d - F_u M(S,I)\|_2^{\,2} + \lambda_S \|\Psi_S(S)\|_1 \qquad (1)$$

where λ_S is a regularization parameter. This non-linear minimization problem can be solved by a two step greedy scheme, which iteratively updates image and coil sensitivities. During the sensitivity update, the minimization problem can be decoupled into subproblems of the form $\min_{S_c} \|d-F_uM_IS_c\|_2^2 + \lambda_S\|\Psi_S(S_c)\|_1$, where M_I is a matrix representing the operator M for a fixed image estimate and S_c is the sensitivity of coil c. In this form, the problem is very close to the Compressed Sensing framework. Two sparsifying transforms Ψ_S were evaluated: a projection on a base of Chebyshev polynomials and the Fourier transform. Coil sensitivities are sparse in these two domains because of their smoothness. The image update can be carried out by Generalized SENSE (GSENSE) [6] using the current sensitivity estimate. In addition, regularization can be applied to penalize the l_1 norm of the image in a certain transform domain in order to limit noise amplification and aliasing artefacts.

Methods: The proposed method was implemented in Matlab. The maximal order for the Chebyshev polynomials was 12. The option to apply regularization on the l_1 norm of the image was implemented for a Daubechies-4 wavelet transform. The image and sensitivity updates were calculated with a non-linear conjugate gradient algorithm as in [7]. In addition, reconstructions with GSENSE and with JSENSE based on polynomials of order 12 were computed for comparison. Spin-echo imaging was performed in the brain on a 1.5T scanner (Achieva, Philips Healthcare) with a custom-made six element head phased array coil (FOV= 250 mm, TE= 5 ms, TR= 1000 ms, slice thickness= 6 mm). A variable density undersampling scheme with outer undersampling factor of 4 and 6 additional autocalibration profiles was applied off-line to yield an overall reduction factor of 3.66. A coil sensitivity estimate obtained from a low resolution scan was used in GSENSE and as initialization in the other algorithms. For JSENSE and for the two proposed methods, convergence was observed after about 10 image and sensitivity updates.

Results: Reconstruction results for the human brain data are presented in Fig.1. The GSENSE reconstruction presents aliasing artefacts due to errors in the low resolution coil sensitivity estimate. JSENSE suppresses most of this aliasing, but is affected by noise. Reconstruction with an l_1 norm constraint on the Fourier transform of the coil sensitivities partially eliminates the aliasing, whereas l_1 norm regularization on the polynomial transform effectively suppresses almost all aliasing artefacts. However, in both cases the final image is also corrupted by noise. Finally, applying wavelet transform l_1 norm regularization in the image update of the two proposed methods yields reconstructions with little traces of aliasing and a significantly reduced noise level.

Discussion and Conclusion: The approach proposed in this work, which is motivated by Compressed Sensing theory, allows solving the ill-posed problem of jointly estimating image and coil sensitivities in parallel imaging. As demonstrated by the results obtained with the proposed method for Chebyshev polynomials and with JSENSE, coil sensitivities are well approximated by polynomial functions. However, methods based on a polynomial representation are very costly from a computational point of view. Applying l_1 regularization on both the wavelet transform of the image and the Fourier transform of the coil sensitivity is a more computationally efficient alternative that also yields images with reduced aliasing and noise. In this case, even for a non-stochastic *k*-space trajectory such as the one used in Fig.1, some degree of incoherence between the sparsity and measurement domains is introduced by the convolution of the sensitivity and the image in *k*-space. This allows for a Compressed Sensing reconstruction, as high frequency components of the image shift part of the sensitivity information away from its original *k*-space location.

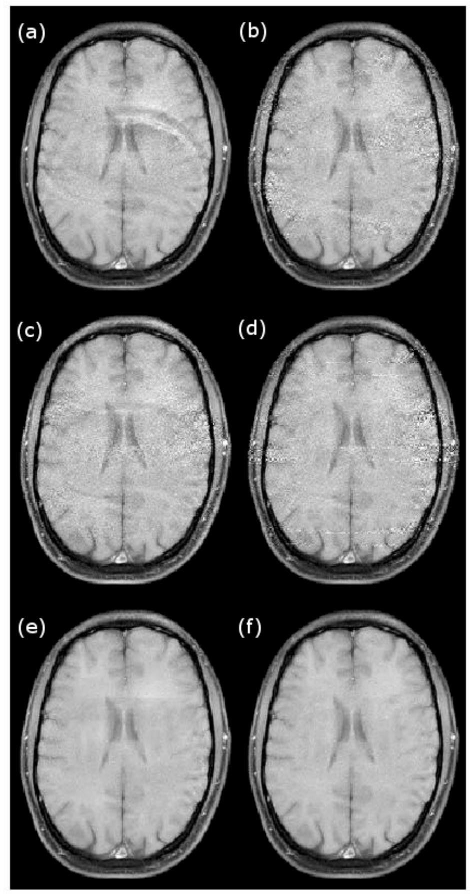

Fig. 1. Reconstruction of brain data for an acceleration factor of 3.66 by GSENSE (a), JSENSE (b), l_1 regularization of the coil sensitivity Fourier transform without (c) and with (e) l_1 regularization of the image norm in a wavelet domain, and l_1 regularization of the coil sensitivity polynomial transform without (d) and with (f) l_1 regularization of the image norm in a wavelet domain.

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