Diffusion propagator imaging: a novel technique for reconstructing the diffusion propagator from multiple shell acquisitions

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INTRODUCTION: Many recent techniques have been introduced for high angular resolution diffusion imaging (HARDI) [1 and references therein], to infer the diffusion or fiber orientation distribution function (ODF) of the underlying tissue structure. These methods, mostly designed for fiber tractography, are normally based on a single shell acquisition and can only recover some angular information contained in the ensemble average propagator (EAP). The (EAP) describes the full three-dimensional (3D) average scattering of water molecules in biological tissue. It can thus provide more information about tissue properties than the ODF. Various methods already exist to estimate the EAP such as [2]-[8], but all have their limitations. Among the most commonly used, diffusion tensor imaging (DTI) [2] is limited by the Gaussian assumption and cannot account for complex fiber configurations, and diffusion spectrum imaging (DSI) [3] is a model-free approach but requires hundreds of diffusion weighted measurements to obtain the EAP. We present diffusion propagator imaging (DPI), a simple and linear analytical EAP reconstruction using Laplace's equation. The solution is computed from only two or more different b-value shells. Robust EAP is possible with less than 100 measurements per shell.

METHODS: If we suppose that Laplace's equation can describe the 3D q-space MR diffusion signal, $E(\mathbf{q}) = S(\mathbf{q}) / S_0$, can be written as,

$$E(\mathbf{q}) = E(q\mathbf{u}) = \sum_{j=0}^{\infty}\left[\frac{c_j}{q^{l(j)+1}} + d_j \cdot q^{l(j)}\right] Y_j(\mathbf{u})$$

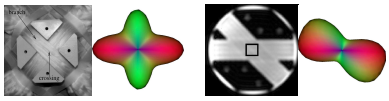
where $\mathbf{q}$ is a 3D vector in q-space, $q = |\mathbf{q}|$, $\mathbf{u}$ is a 3D unit vector, Y_j is the modified even, symmetric, real, and orthogonal spherical harmonic (SH) basis [1] and l(j) is the order associated with the jth element of the SH basis. Boundary conditions need to be given to solve for the coefficients, which in our problem are the signal without diffusion gradient S_0 (when q = 0) and at least two different b-value shell measurements. Intuitively, this can be thought as the heat equation between each shell. A priori, there is no physical reason why this should be so but the spherical Laplace equation can model the diffusion signal satisfactorily and allows one to solve for the EAP analytically. Under the narrow pulse assumption, the relationship between the diffusion signal attenuation and the EAP is given by a Fourier transform (FT), $P(\mathbf{R}) = FT[E(\mathbf{q})]$. After some algebra and using properties of the spherical harmonics and spherical Bessel functions, we can write the EAP as

$$P(R_0\mathbf{r}) = \sqrt{\frac{8\pi}{R_0}}\sum_{j=0}^{\infty}\frac{(-1)^{l(j)/2}(2\pi R_0)^{l(j)-3/2}}{(2l(j)-1)!!}\cdot c_j Y_j(\mathbf{r}), \quad \text{where } (n-1)!! = (n-1)\cdot(n-3)\cdot\ldots\cdot 3\cdot 1$$

$P(R_0\mathbf{r})$ represents the probability of finding a water molecule at distance $R_0\mathbf{r}$ for the origin. Hence, it can be viewed as the EAP values on a sphere of radius R_0. Note that the DOT [5] reconstruction is similar in spirit but starts from a mono or bi-exponential decay assumption of $E(\mathbf{q})$. Here, our solution has an analytical form and is linear. It only depends on the c_j coefficients. The d_j coefficients have disappeared in the EAP reconstruction. However, these d_j are needed for accurate $E(\mathbf{q})$ estimation.

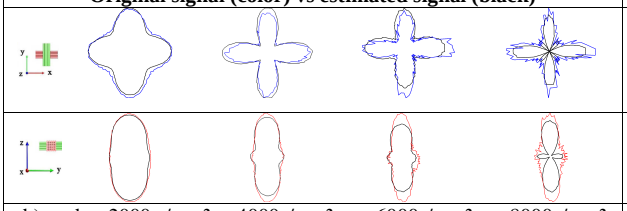
RESULTS: The proposed method was used to estimate the diffusion signal and the associated EAP on ex vivo phantoms [9] with fibers crossing at 90° and 45°, using a a 1.5T system, TE/TR=130ms/(4.5s,12.0s) (45°and 90°), BW=200KHz and b-values of 2000, 4000, 6000, 8000 s/mm^2, with 4000 uniformly distributed orientations.

a)



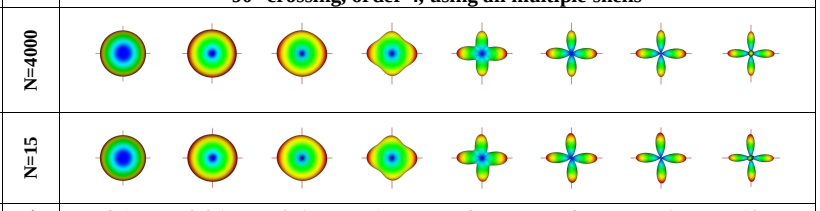
	90° crossing, sampling scheme N					45° crossing, sampling scheme N				
Shells	N=4000	N=60	N= 40	N=25	N=15	N=4000	N=60	N=40	N=25	N=15
2000	2.6%	2,7%	2.6%	2,6%	2.8%	2.1%	2.1%	2.2%	2.1%	2.3%
4000	4.3%	4.3%	4.3%	4.3%	5.1%	4.0%	4.0%	4.0%	4.0%	4.0%
6000	5.1%	5.2%	5.2%	6,2%	6.2%	5.8%	5.8%	5.8%	5.8%	6.3%
8000	8.5%	8.5%	8.5%	8.6%	8.6%	8/5%	8.5%	8.5%	8.5%	8.5%

Original signal (color) vs estimated signal (black)



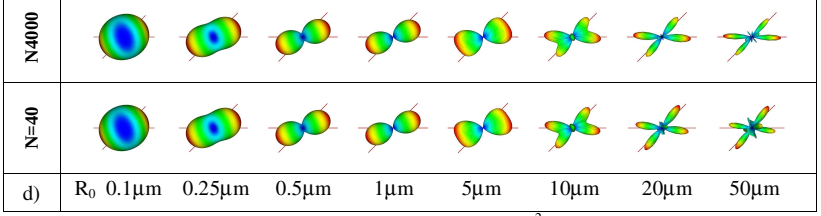
b) b = 2000 s/mm² 4000s/mm² 6000s/mm² 8000s/mm²

90° crossing, order 4, using all multiple shells



c) R_0 0.1µm 0.25µm 0.5µm 1µm 2µm 3µm 5µm 10µm

45° crossing, order 6, using two outmost shells b =2000 and 8000 s/mm



d) R_0 0.1µm 0.25µm 0.5µm 1µm 5µm 10µm 20µm 50µm

Figure a) illustrates the 90° and 45° phantoms with associated diffusion ODF from q-ball imaging [1]. In b), the original diffusion signal (color) and estimated signal (black) of order 4 are compared using all four available shells. In c), the table quantitatively illustrates the percentage error between original and estimation signal for different sampling scheme N. As seen in b) and the table, estimation is accurate and remains robust even for lower gradient sampling schemes, such as N = 15 to 60 measurements per shell. Figures c) and d) show the associated EAP reconstructions for different radius R_0 at 90° and 45°. c) uses all four shells and SH order 4 with, on top, all N = 4000 directions per shell, and below, only N=15 directions per shell. d) uses only the two outmost shells (b=2000 and 8000 s/mm^2) and SH order 6 with, on top, all N=4000 directions per shell, and below, only N=40 directions per shell. The 90° and 45° crossing configuration is clearly distinguishable for high R_0 compared to the classical single shell q-ball ODF reconstruction. Having an analytical solution also allows one to estimate the propagator for any R_0, i.e. higher than maximum q-value.

DISCUSSION: This work shows two important contributions. 1) The q-space diffusion signal can be modeled using spherical Laplace's equation. 2) The EAP can be estimated analytically with a simple and linear solution. The dsignal estimation and EAP reconstruction were validated on ex vivo phantoms. Using four shells in q-space with only 15 diffusion measurements per shell, the signal and EAP of SH order 4 were reconstructed as robustly and precisely as when using 4000 measurements per shell. Moreover, using only the two outermost shells in q-space (2000 and 8000 s/mm^2 shells), we were able to reproduce similar EAP reconstructions. Finally, low separation angles, such as in the 45° phantom, can be distinguished for high R0, if one uses a SH order of 6 or higher. Therefore, it seems promising that DPI acquisitions with less than 100 diffusion measurements, from two or more shells, can be sufficient to robustly reconstruct the diffusion propagator on the human brain.

References: [1] Descoteaux, PhD Thesis, 2008. [2] Basser et al, JMR B, 1994. [3] Wedeen et al, ISMRM, 2000. [4] Liu et al MRM, 2004. [5] Ozarslan et al, NeuroImage, 2006. [6] Lu et al, NMR Biomed, 2006. [7] Pickalov & Basser, ISBI, 2007. [8] Ozarslan et al, ISMRM. 2008. [9] Poupon et al, MRM, in press, 2008.