# Joint Design of Excitation and Refocusing Pulses for Fast Spin Echo Sequences in Parallel Transmission

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### INTRODUCTION

Fast spin echo (FSE) sequence [1] has been widely used in various MR applications, which, like other sequences, can suffer from the inhomogeneous  $B_1$  at high field. Parallel transmission of RF pulses [2, 3] has been shown to be quite effective in correcting  $B_1$  inhomogeneity. However, no FSE application with parallel transmission has been seen in the literature yet primarily due to the lack of method for designing parallel transmit pulses for FSE that satisfy the CPMG condition [4]. In this paper, we first show that designing excitation and refocusing pulse pair for FSE is a joint design problem. Then we formulate the design problem as a novel optimal control problem, where the optimal pulse pair is sought jointly to drive the magnetization vector to its desired state while maintaining the CPMG condition. Bloch simulation results are shown to demonstrate the superior performance of the pulses designed by the proposed method to those from conventional methods.

#### PROPOSED METHOD

As all 180 degree pulses in an FSE sequence are usually identical, it is sufficient to consider a 90 degree excitation pulse followed by a single 180 degree refocusing pulse with the constraint that the phase angle of the nutation axis at spatial location  $\mathbf{r}$  for the excitation pulse  $\theta(\mathbf{r})$  needs to lead (or trail) that for the refocusing pulse by 90 degrees. In parallel transmission,  $\theta(\mathbf{r})$  is in general not constant over space due to the complex nature of the  $B_1+$  map and is typically treated as a design parameter [5]. The pulse design problem is therefore to choose  $\theta(\mathbf{r})$  such that the excitation pulse with its nutation axis phase being  $\theta(\mathbf{r})+90$  drive the magnetization as close to its targeted profile as possible. Denote the durations of the excitation and refocusing pulses to be  $t_1$  and  $t_2$ , respectively. To ensure the refocusing pulse can refocus a collection of spins with different initial phases of transverse magnetizations, it is sufficient to consider only two magnetization vectors  $\mathbf{m}(t)$  and  $\mathbf{n}(t)$  where  $\mathbf{m}(t_1)$  is the magnetization right after the excitation pulse is applied and the transverse component of  $\mathbf{n}(t_1)$  is 90 degrees ahead of  $\mathbf{m}(t_1)$ . The desired transverse components of  $\mathbf{m}(t_1)$  and  $\mathbf{n}(t_1)$  have magnitude 1 everywhere (normalized to equilibrium magnetization) and phase angles  $\theta(\mathbf{r})+90$  and  $\theta(\mathbf{r})+180$ , respectively. Note due to CPMG condition, the phase of the nutation axis for the refocusing pulse is  $\theta(\mathbf{r})+90$  as well. Therefore,  $\mathbf{m}$  is expected to remain as  $\mathbf{m}(t_1)$  and  $\mathbf{n}$  is expected to be flipped to  $-\mathbf{n}(t_1)$  at the end of the refocusing pulse  $t=t_1+t_2$ . The joint design problem can therefore be formulated as choosing the L-channel RF pulses  $b_1(t_1),...,b_L(t)$  (where  $0 < t < t_1$  is the interval for the excitation pulse and  $t_1 < t_1 + t_2$  is for the refocusing pulse) to

$$\mathbf{Minimize} J = \sum_{\mathbf{r}} \left[ \left( |m_{xy}(t_1)| - 1 \right)^2 + m_z^2(t_1) + \left( |m_{xy}(t_1 + t_2)| - 1 \right)^2 + m_z^2(t_1 + t_2) + \left( |n_{xy}(t_1 + t_2)| - 1 \right)^2 + n_z^2(t_1 + t_2) + \left[ |m_{xy}(t_1 + t_2) - m_{xy}(t_1)|^2 + [n_{xy}(t_1 + t_2) + n_{xy}(t_1)]^2 \right] + \lambda \sum_{l=1}^{L} \int_{0}^{t_1 + t_2} \left| b_l \right|^2 dt, \quad (1)$$

where subscripts xy, and z denote the transverse and longitudinal components of  $\mathbf{m}$  and  $\mathbf{n}$ . The first term of Eq. (1) enforces the desired conditions at  $t = t_1$  and  $t = t_1 + t_2$ . Note the  $\theta$  dependency is implicitly included. The second term is an RF power regularization term with  $\lambda$  being the weighting factor balancing the two terms. Both  $\mathbf{m}$  and  $\mathbf{n}$  are subject to the Bloch equation. The above formulation is an optimal control formulation and can be converted into a 3-point boundary-value problem [6]. A first order gradient descent method is implemented to obtain the optimal  $b_l(t)$  numerically. Implementation details are omitted for limited space.

#### RESULTS

Figure 1 shows the Bloch simulation results of three sets of excitation and refocusing pulses in dual channel transmission for  $B_1$  inhomogeneity correction. The  $B_1$ + maps are acquired using a dual channel transmission 3T GE Signa scanner with a torso phantom (boundary of the phantom shown as white dotted line). A 3-turn unaccelerated inherently refocused spiral trajectory [7] is used to cover the excitation k-space for both excitation and refocusing pulses. In addition to  $\theta(\mathbf{r})$  (column 1), three other quantities (columns 2 to 4) are also plotted as a measure of the excitation and refocusing performance. Note a well-designed pulse pair would generate close to zero results for all three quantities. We design the first set of pulses by separately designing excitation and refocusing pulses using the conventional optimal control design [8] and enforcing a predetermined  $\theta$  (the quadrature phase of the body coil in this simulation) for the excitation and  $\theta$ +90 for the refocusing pulses (method a). This design generates noticeable residual signals for all three quantities as a result of the heuristic nature of  $\theta$  choice. We design the second pulse by choosing the optimal  $\theta$  for the excitation pulse using the phase-relaxed design in [5], and then enforce the constraint of the nutation axis pointing in  $\theta$ +90 in designing the refocusing pulse. As seen in Fig. 1b, the excitation profile is significantly improved over that from method a due to optimization on  $\theta$ . However, the resulting  $\theta$  is not "compatible" with the refocusing pulse design and generates larger residual signals after applying the refocusing pulse. We design the third pulse using the proposed joint design. As shown in Fig. 1c, the joint design leads to greatly improved excitation and refocusing profiles over the first two methods. Detailed sum of squared errors of the quantities in Fig. 1 and other quantities are listed in Table 1.

## REFERENCES

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	θ(r)	$ m_{xy}(t_1)  - 1$	$m_{x}(t_1+t_2)-m_{x}(t_1)$	$n_x(t_1+t_2)+n_x(t_1)$
(a) Separate Design, Prechosen θ(r)				
(b) Separate Design, θ(r) Optimized for Excitation Pulse				
(c) Joint Design, θ(r) Optimized for Both Pulses				
	-180 0 180 (degree)	-0.3 -0.2 -0.1 0(× M <sub>o</sub>	-0.5 0 0.5 (× M <sub>o</sub> )	-0.5 0 0.5 (× M <sub>o</sub> )

Fig. 1. Bloch simulation of dual channel transmission of three sets of excitation and refocusing pulse pairs designed by three methods for  $B_1$  inhomogeneity correction. The four columns are nutation axis phase angle for the excitation pulse, error of the transverse magnetization after the excitation pulse, error of the  $m_x$  component and error of  $n_x$  component after the refocusing pulse, respectively. The three design methods are: a) Separate design of excitation and refocusing pulses with a given  $\theta$ , b) Separate design where  $\theta$  is optimized for excitation pulse and  $\theta$ +90 is enforced in designing refocusing pulse, and c) the proposed joint design of the two pulses. See text for detailed discussion.

	Method a	Method b	Method c
$ m_{xy}(t_1) $ - 1	0.06	0.03	0.04
$m_z(t_1+t_2)$	32	113	8
$n_z(t_1+t_2)$	43	128	11
$m_{\scriptscriptstyle X}(t_1+t_2)-m_{\scriptscriptstyle X}(t_1)$	14	182	0.7
$m_{y}(t_1+t_2) - m_{y}(t_1)$	7	105	0.8
$n_x(t_1 + t_2) + n_x(t_1)$	9	170	0.7
$n_{y}(t_{1}+t_{2})+n_{y}(t_{1})$	13	104	1.6

Table 1. Sum of squares ( $l_2$  norm) of various quantities that measure the performance of an excitation and refocusing pulse pair.