Sparse Parallel Transmit Pulse Design Using Orthogonal Matching Pursuit Method

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Introduction Parallel RF transmit (Tx) offers additional degrees of freedom (redundancy) in excitation pulse design [1,2]. One application of this redundancy is accelerating multidimensional selective excitation by under-sampling the excitation k space. Unlike the spatial encoding task in imaging, where the underlying image is unknown, in the case of excitation pulse design the target excitation profile is known *a priori*. This provides a Bloch equation-based RF pulse design with great opportunities for optimization. In principle, the target profile, B1 maps and error tolerance (TOL) determine where to ideally under-sample the k space with optimal sparsity. It is desirable to have a sparsifying strategy that exploits this pre-knowledge [3,4]. We propose a k-space sparsifying method based on a greedy-wise algorithm. Our approach is inspired by the theoretical results in sparse signal approximation [5,6,7] and the Compressed Sensing (CS) effort on the imaging side [8,9]. In the sense of 'sparsity' our approach can be seen as a counterpart to the multi-coil CS in RF excitation. Simulation and phantom results showed good excitation profile particularly in high reduction-factor cases, where the conventional non-adaptive under-sampling method is limited.

Theory Parallel Tx pulse design is described in a matrix notation by $p = \Sigma_j \mathrm{Diag}(S_j)^*\Phi_{FT}{}^*b_j$ where p is the target profile, b_j is the rf waveform for the j-th Tx-coil, S_j is the B1 map of the j-th Tx-coil, Φ_{FT} is the Fourier Transform matrix: $[\varphi_{k1,\dots,}\varphi_{kN}]$, where $\varphi_{kj} = \exp(ik_jx)$ is the Fourier base for j-th Nyquist k location. The goal of parallel Tx adaptive sparse k/RF joint design is to represent the target p by using fewest k locations as possible given TOL. This perspective leads us to sparse approximation with redundancy [5,6,7], which can be approached either by solving L1-regularized-LS problem [7,4] , or by greedy style method [5,6], where one stepwise introduces the next 'most important' k point. If in Fourier domain the target profile and the B1 maps have sparse representations, one can end up with a much smaller number of k locations than the number of k locations required by Nyquist theorem. For parallel Tx acceleration one needs s*imultaneous* sparsity over all the channels [4], which makes the greedy approach a more suitable choice.

Method Our proposed method is a modified version of the so-called Orthogonal Matching Pursuit (OMP) [5,6] method. (Notations: N = number of spatial pixels, N_c = the number of channels, N_k = the number of k locations)

Step-I: Find the next 'best' k location:

$$k_{\text{next}} = \underset{k \in \text{Nyq. grid}}{\operatorname{argmin}} \; (\|\text{res} - A(k)\cdot b\|_2) \;\; \text{for any } b \in \mathbb{C}^{N_C}$$

where A is matrix composed of N_c column-wise B1 weighted Fourier harmonics:

$$A(k) = [S_1 \cdot e^{ikx}, \dots, S_{N_c} \cdot e^{ikx}] \in \mathbb{C}^{N \times N_C}$$

Step-II: Add the new k location to the previously chosen k subset

Step-III: Use LS pulse design for the best possible profile with chosen k positions. Calculate the corresponding residual: $\text{res} = \Sigma_j S_j{}^*\Phi_{FT}{}^*b_j - p$

Step-IV: *if* $|\text{res}|^2 > \text{TOL}$, → step-I)
else , → step-V)

Step-V: Design the feasible k trajectory traveling through $k_{1:N_k}$

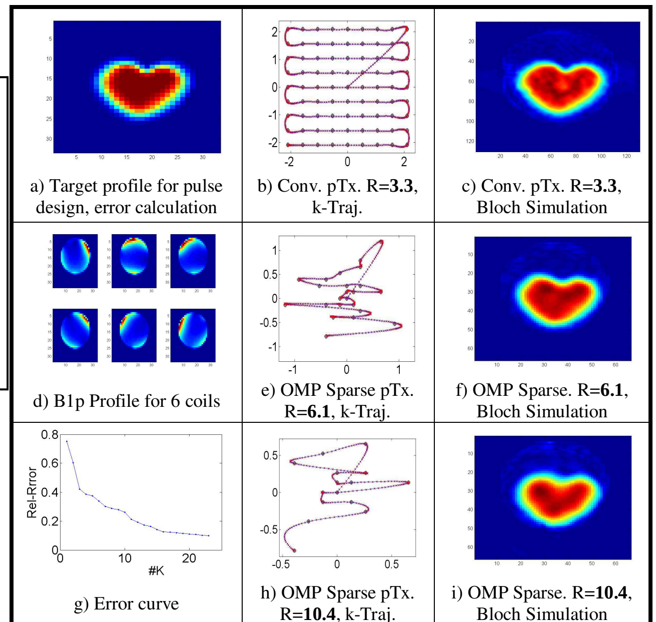
Step-VI: Redesign RF with the resulting k-trajectory using conventional pulse design.

The sparsity of the k space is related to the smoothness of the B1 spatial profile. To avoid unnecessary spatial singularities we apply a smoothly decaying-to-zero extrapolation of the B1 profiles into the don't-care region outside the imaged object. Calculation of a k trajectory that efficiently traverses an arbitrary set of k space locations is non-trivial. We currently use an approach that connects the locations in a suboptimal EPI-like manner. For gradient waveform design we use (i) a routine based on [11] or (ii) a suboptimal extended version based on [10].

Results We validated the adaptive OMP parallel Tx method using Bloch simulations and phantom experiments. Fig.1 shows the simulation results using 6 Tx Coils. Fixed under-sampling strategy delivered reasonable results at a moderate reduction factor (R=3.3 with 6 Ch.). With higher R the quality is limited. Adaptive OMP Sparse method produced good results at very high reduction factor (Fig1f,i). Fig.1g shows the error curve as a function of the number of k locations. Fig.2 shows phantom MRI results obtained on a GE 3T scanner using two parallel transmit channels (d). With R number of channels (**R**=2.0,2.8) we still achieved reasonable excitation profiles.

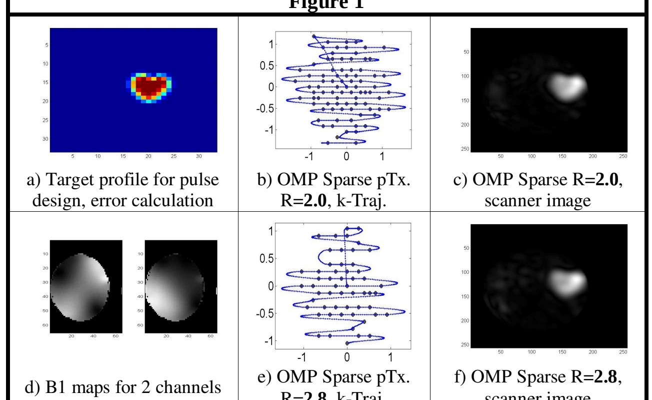
Discussions The foundation of this work is: A) Parallel Tx technology offers additional redundancy, and B) Practical selective excitation target profiles do not require full sampling of a Nyquist-k-grid to start with. Based on both A) and B), we have developed a method that under-samples the k space with pursuit of maximum sparsity, which promises to significantly reduce the excitation pulse duration beyond the limit of conventional parallel Tx. This method allows also a flexible tradeoff between the excitation quality and the pulse duration. The method was validated in Bloch simulations and phantom imaging studies.

We note that the B1 map profiles are essential for how sparse a pulse can be – smoother profiles tend to allow greater sparsity. This method can be used to help determine the number and the positions of the spokes for 3D spokes-pulse design. In the current form the phase-relaxing mechanism [12] is only involved in the error calculation step. We expect further significant improvement of the sparsity by possibly integrating it also into the OMP greedy steps.



a) Target profile for pulse design, error calculation; b) Conv. pTx. R=**3.3**, k-Traj.; c) Conv. pTx. R=**3.3**, Bloch Simulation; d) B1p Profile for 6 coils; e) OMP Sparse pTx. R=**6.1**, k-Traj.; f) OMP Sparse. R=**6.1**, Bloch Simulation; g) Error curve; h) OMP Sparse pTx. R=**10.4**, k-Traj.; i) OMP Sparse. R=**10.4**, Bloch Simulation

Figure 1



a) Target profile for pulse design, error calculation; b) OMP Sparse pTx. R=**2.0**, k-Traj.; c) OMP Sparse R=**2.0**, scanner image; d) B1 maps for 2 channels; e) OMP Sparse pTx. R=**2.8**, k-Traj.; f) OMP Sparse R=**2.8**, scanner image

Figure 2

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