Sequential Optimal Spoke Selection for Spoke Trajectory Based RF Pulses Design in Parallel Excitation

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INTRODUCTION: The spoke trajectory [1,2] is a preferred choice of k-space trajectory in designing 3D RF pulses for applications requiring thin slice selection and tailored in-plane excitation. It typically consists of several spokes of samples, each spoke extensively covering the kz (through-plane) axis at a distinct (kx,ky) (in-plane axes) location. Ideally, a large number of spokes should be used to ensure sufficient coverage of the kx-ky plane. In practice, however, only a small number of spokes can be used due to the pulse duration limitation. The locations of these spokes are therefore critical to the performance of the resulting RF pulses and should in principle be optimized for a given excitation pattern and B1 field map, especially in parallel transmission. In this paper, we propose a sequential selection [3,4] based algorithm to find the optimal sampling locations that generates minimum excitation error. Both Bloch simulation and experimental results show that the proposed method produces significantly smaller excitation error and/or shorter pulses than several conventional designs.

PROPOSED METHOD: Suppose there are L transmit coils and N spokes whose locations $K_N=\{(k_{n,x}\ k_{n,y})\}$, $n=1,\ldots,N$ are given, Sinc/SLR subpulses are used in the 3D design and the weights are determined by minimizing the following cost function:

$$\|A(K_N)\beta - d\|_2^2 + \lambda\|\beta\|_2^2, \beta \in \mathbf{C}^{NL\times 1}, A(K_N) \in \mathbf{C}^{N_s\times NL}, d \in \mathbf{C}^{N_s\times 1}, \quad (1)$$

where β is a vector of subpulse weights; d is a vector containing samples of the desired excitation pattern; $A(K_N)$ is a system matrix depends on spoke locations and sensitivity maps of transmit coils; and λ is used to trade off the excitation error and the RF power. Closed form expression of the minimum of (1) exists, and is given by:

$$J(K_N) = d^H d - d^H A(K_N)\left(A^H(K_N)A(K_N) + \lambda I\right)^{-1} A^H(K_N)d, \quad (2)$$

which explicitly depends on the choice of spoke locations. Therefore our goal is to select optimal spoke locations that give the smallest value of (2).

The optimization problem is a combinatorial optimization problem, whose computation complexity is not acceptable. Hence, a greedy method is proposed to solve the problem efficiently. Starting from an empty selection, one more spoke is sequentially selected at each step, while maintaining the previous spoke selection. At each step, the spoke that maximally reduces the cost function value is selected. The procedure stops when a predetermined number of spokes are selected. The major computation burden comes from evaluating the inverse $(A^H(K_N)A(K_N)+\lambda I)^{-1}$ in (2). The computation complexity for computing this inverse directly is in the order of $O(n^3L^3+n^2L^2N_s)$, where n is the number of spokes at the previous step. However, notice that when one more spoke is selected, L new columns and L new rows are added to the matrix $(A^H(K_N)A(K_N)+\lambda I)$. The inverse can be evaluated recursively, which reduces the computation complexity of evaluating (2) to the order of $O(nL^2N_s)$ with necessary pre-calculations.

RESULTS: RF pulse design and the spoke selection algorithm were implemented with MATLAB 7.1 (MathWorks, Inc., Natick, MA), on a 1.7 GHz Pentium M laptop with 1 GB of memory. The desired excitation pattern was a circle filtered by a Hamming filter in a 36×36 cm FOX. The FWHM is 7.9 cm. The slice thickness was 10 mm. The candidate grid of spoke locations was a Nyquist-spaced 18×18 square grid. Five different methods were compared: 1) Fourier method: spoke locations are chosen based on the discrete Fourier transform of the desired excitation pattern, and locations with largest magnitudes of Fourier coefficients are chosen; 2) Cartesian-spiral method: spoke locations are chosen based on a Cartesian spiral trajectory; 3) inversion method [5]; 4) sparsity-enforced method [5]: as in ref. [5], the SeDuMi package [6] was used; 5) the proposed method.

Bloch simulation was first implemented to compare different methods. The excitation error in percentage as a function of the number of spokes, is shown in Fig 1. The proposed method produces the smallest excitation error in all cases. Suppose the excitation error is required to be below 1%. Using the proposed method 13 spokes or 5.3 ms RF pulse are needed, compared with 23 spokes or 9.7 ms RF pulse for the sparsity enforced method.

Experiments were performed using a gradient-recalled echo (GRE) sequence, implemented on 3T GE Excite scanner (GE Healthcare, Milwaukee, WI, USA). Two channel body coils were used for transmission and receiving. Data acquisition parameters were: TE = 10 ms, TR = 100 ms, flip angle =30°. The number of spokes was chosen to be 15, leading to 7 ms RF pulses. Experimental results are summarized in Fig 2. Compared with other methods, the proposed and sparsity-enforced methods improve the excitation accuracy significantly. However, the proposed method has higher computation efficiency. In current implementation, the computation time of the proposed method is about 7s, compared with about 6 minutes for the sparsity enforced method for single regularization parameter. Spoke locations selected by different methods are shown in Fig 3.

CONCLUSION: A new algorithm was presented to select optimal spoke locations for spoke trajectory based slice RF pulses in parallel excitation. The optimal spoke selection problem was formulated as a combinatorial optimization problem, which was efficiently solved using a greedy algorithm with recursive evaluation of the cost function. Simulation results and experimental results on a 3T GE Excite scanner with two-channel parallel excitation system showed that the proposed method outperformed existing methods with improved computational efficiency.

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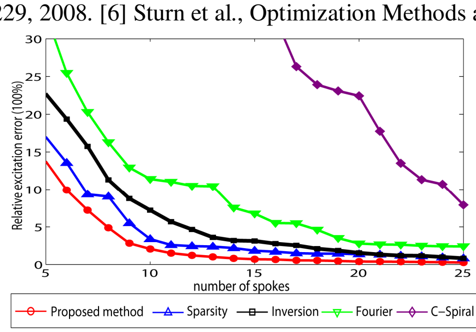

Fig. 1: Bloch simulation, relative excitation error vs. # of spokes.

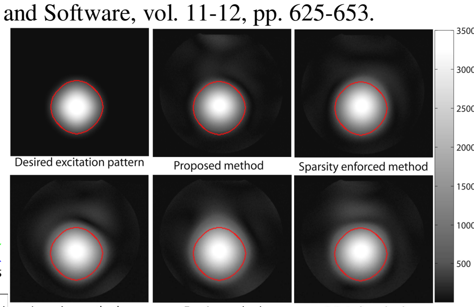

Fig. 2: 3D excitation patterns. The red circle represents the boundary of the desired excitation pattern.