Voxel Based Topometry of the ADC Profiles: Collapsing of the Dimensionality

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1. Introduction: Diffusion is a phenomenon which allows one to look inside the voxel via the macroscopical characteristics. These characteristics comprise a large number of inputs of the individual water molecules travelling in the heterogeneous brain structure. The trajectories are rather complex but in the average sense they are correlated with the architecture of the brain. We explore the property of the magnetic resonance diffusion attenuated signal distributed in 3D space over the truncated icosahedra scheme and introduce a set of geometrical measures and their topological characteristics – non-integer dimensions. These indices reflect the complex nature of the diffusion signal and capture the spatial anisotropy of the brain within the voxel. Our findings suggest that non-Gaussian distribution of the diffusion signal dominates in the brain and carries more detail information which is complementary to the diffusion tensor approach [2].

2. Methods: 67 images were acquired for a single slice on a Siemens 3T Trio scanner using a double-refocused, diffusion-weighted spin echo sequence (TR/TE=10000/88ms, voxel size=(1.8 mm)^3, b-value=800s/mm^2) from a healthy male adult. Sixty diffusion-weighted images, corresponding to the sixty directions of diffusion gradients distributed according to the truncated icosahedra scheme, were used. Additionally, seven images with b-value=0 were randomly acquired between diffusion-weighted images to factor out the T_2–weighting of the signal. The scan was repeated four times in a single session (without removing the subject from the scanner). The *ADC* profiles were recovered using Eq. 1 and afterwards interpolated for voxels within and outside the brain. The profiles demonstrated very complex structure, correlated with the anatomical architecture. In the Fig.1a the T_2-weighted image of the transverse slice is shown and *ADC* profiles are demonstrated in Fig.1b-d for various voxels. The structure of the profiles can be analysed by the modified iterative scaling *blowing sphere method* [3]. The key point of this method consists in the step-by-step increase of the radius of the sphere over all icosahedra directions from the minimal to the maximal value. During expansion of the sphere in the certain direction the growth was stopped if the radius of the sphere exceeded the acquired *ADC* profile boundary in a given direction. In the Fig. 2a, the 3D process is depicted with some intermediate steps in the Fig. 2b-c (in these figures the equatorial cross sections of the *ADC* profiles are presented).

Fig.1a **Fig.1c** **Fig.1d** **Fig.1b**

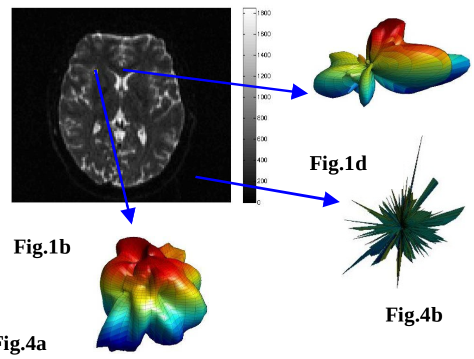


Fig.2a

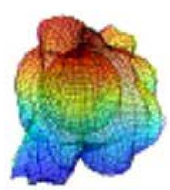


Fig.2b

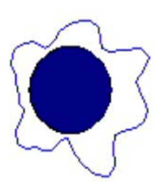


Fig.2c

Fig.3

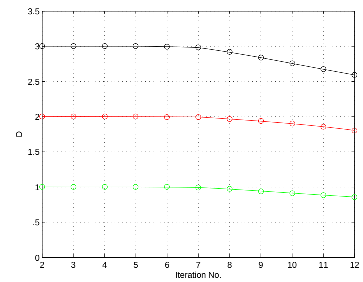


Fig.4c

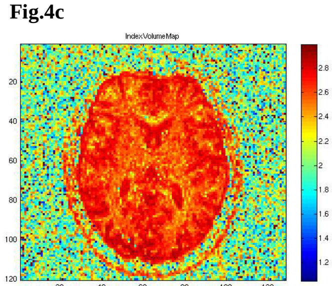


Fig.4a

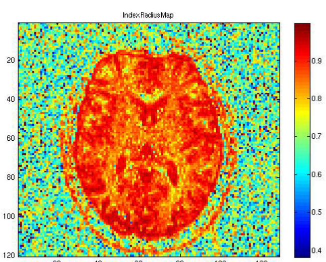
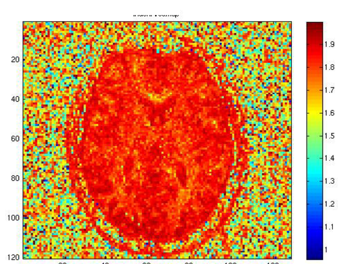


Fig.4b



$$S/S_0 = e^{(-b \cdot ADC)} \qquad (1)$$

$$M\big|_{M=V,S,R} \sim R_i^D \qquad (2)$$

Fig.1a. T_2-weighted image with randomly selected voxels.

Fig.1b-c. *ADC* profiles within the brain area. These profiles demonstrate correlation with anatomical structure of the brain. (b) grey matter; (c) white matter.

Fig.1d. *ADC* profile outside the brain area. This profile exhibits the strong noisy structure.

Fig.2a. 3D *ADC* profile with blowing sphere inside.

Fig.2b-c. 2D *ADC* profile equatorial cross section with blowing sphere projection inside: (b) sphere has not reached the *ADC* profile border; (c) sphere starts to be corrupted by the *ADC* profile border.

Fig.3. Collapsing of the dimensionalities (Eq. 2). At the initial steps of iterations *D*–power corresponds to the dimensions of the Euclidean space.

Fig.4a-c. The maps of the collapsed dimensions of: (a) mean radius; (b) surface area; (c) volume.

3. Results: We calculated the volume *V*, surface area *S* and mean radius *R* during blowing sphere iterative scaling process and compared it to the spherical measures by adjusting the power *D* of the mean radius R_i of the surface (Eq. 2, *i* is an iteration number). The scaling iterative procedure of *D*–power changes (Fig. 3) exhibits three regimes of behaviour: stable regime (plateau), transition and linear collapsing regime. The linear collapsing regime approaches the attractor points corresponding to the random surface (*ADC* profile). In the stable regime for the volume, surface and mean radius values *D*–power was 3, 2 and 1 correspondently. These are the dimensions of the initial, non-corrupted iteratively blowen sphere. After passing the nonlinear transition regime of collapsing of dimensions, the *D*–power achieves homogeneously decaying state. The maps of the collapsed parameters (attractor points of *D*–power) are presented in Fig. 4a-c.

4. Discussion: We suggest a method to characterize the complex structure of the *ADC* profile. The key point of the method consists in iterative scaling of the blowing sphere. With the use of the proposed method, a new set of parameters were mapped. These parameters characterise the topology of the *ADC* profile and correlate with the anatomical structure of the brain. The method provides a novel approach that may enhance the utility and specificity of the diffusion-weighted *MRI* to better assess the structural changes that occur during development and various neuropathologies.

References: [1] Callaghan P, Principles of Nuclear Magnetic Resonance Microscopy (Clarendon Press, Oxford) 1991. [2] Basser P, Mattiello J, Le Bihan D, Biophys. J. **66** (1994) 259. [3] Poznansky O. P., Novikov V. V., J. Polym. Eng. **19** (1999) 223. [4] Mandelbrot B, Fractal Geometry of Nature (Freeman) 1982.