Denoising HARDI Coefficients using Spherical Wavelet Lifting

S. Olhede[1], and B. Whitcher[2]

[1]Department of Statistical Science, University College London, London, United Kingdom, [2]Clinical Imaging Centre, GlaxoSmithKline, London, United Kingdom

Introduction

Depending on the choice of b-value and voxel size the noise floor of the Rician distribution becomes problematic when estimating the diffusion density. We propose to estimate the diffusion distribution at each voxel by using discrete wavelet coefficients on the sphere, combined with an estimation procedure that is adapted to random variables from the Rician distribution. Current non-parametric estimation methods are based on smoothing with a fixed bandwidth; such as the scaling parameter of radial basis functions or the number of basis functions used in PAS-MRI [1], and fixed bandwidth smoothing may remove features of interest in the diffusion PDF. Our procedure takes advantage of an adaptive bandwidth at multiple scales and respects the sampling of gradient-encoding directions on the sphere. This will enable the estimation of fine details in for example white-matter microstructure. We report the mean square error (MSE) in Q-space and calculate the orientation distribution function (ODF) for both simulated and clinical data, respectively.

Methods

To calculate a wavelet transform on the sphere, one must interpolate the data or use a genuinely discrete transform. To avoid smoothing out details in Q-space we construct a discrete wavelet transform using lifting [2]. Wavelet coefficients are calculated using the nearest measurement on the sphere (distances are based on great circles). We use the translation-invariant transform, and do not sub-sample the coefficients at each scale. The transform is terminated at a given scale, called the Primary Resolution (PR). The variance of the noise is robustly estimated from the high-frequency wavelet coefficients using a maximum absolute deviation (MAD) estimator. The scaling coefficients are corrected by removing the bias introduced from the Rician noise floor, using the estimated variance and the raw scaling coefficients. The wavelet coefficients are thresholded using a universal thresholding rule, adapted for Rician random variables, and which uses a local estimator of the variance. After thresholding, the wavelet transform is inverted, and the Q-space density estimated. We implement the approximate Funk transform [3] to estimate the ODF, thus avoiding an explicit usage of a smoothing bandwidth.

Results

Simulated data were generated based on the Gaussian model, for 60 directions, at SNRs and tensors (p_0 and p_3 with a=0.5) proposed in [1]. The performance is reported in average mean square error (MSE) as well as average distance between the closest true and maximum estimated peak for the single tensor; see Table 1. The reduction in MSE is appreciable for both the single and double tensor simulations, corresponding to halving the mean MSE. The variance of the MSE is reduced to a third or tenth of the noisy MSE for some of the investigated SNRs. For single tensor (p_0), the estimated peak orientation is better using the smaller choice of PR, despite the MSE of the estimated density being larger. For the double tensor (p_3) the higher PR outperforms the lower in both MSE and mean distance to the closest true peak (Table 1).

Table 1: (p_0/p_3)	Av. MSE, SNR=1/16	Av. MSE, SNR=1/10	Av. Orient Dist, SNR=1/16	Av. Orient Dist, SNR=1/10
Noisy	1.5482/4.7240	1.7885/4.9328	0.1302/0.1818	0.1345/0.1878
Smooth (PR 1)	0.8620/2.6041	0.9879/2.7096	0.1005/0.1494	0.1046/0.1570
Smooth (PR 2)	0.7210/1.9746	0.7836/2.0384	0.1125/0.1237	0.1149/0.1272

Discussion

The smoothing of the Q-space measurements produces a clear reduction in the MSE. The estimated dominant orientation is also showing a clear reduction in average error for both SNRs. Note that unlike [4] we have applied a suitable thresholding procedure for the wavelet transform of the magnitude data, not assuming the complex-valued object is retained. We show plots of estimated fiber structure from a clinical acquisition [5]. We estimate the ODF for a voxel in the splenium of the corpus collosum, and at the intersection of the corticospinal tract and the superior longitudinal fasiculus [Fig. 1]. The smoothed density of the unidirectional voxel exhibits greater symmetry, while the potentially multi-directional voxel produces a higher-resolution estimate, see Figure 1. The wavelet estimates of the Q-space distribution do not have a single associated smoothing bandwidth, but rather the smoothing level is different at each coefficient. This allows a flexible reconstruction of signals, potentially allowing us to determine local intra-voxel structure, where summaries of the distribution can be used in for example tractography. More flexible estimators of diffusion density are required as FA alone does not reflect multiple changes of radius/orientation and/or permeability [6].

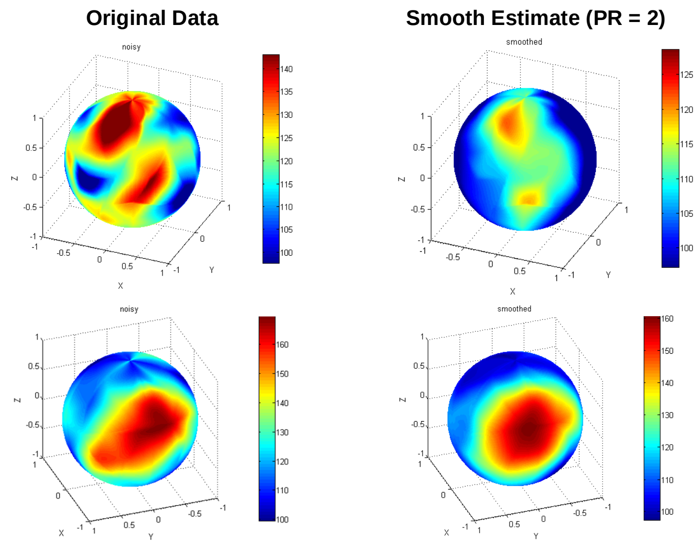

Figure 1: *Estimated ODF from the data in [4] for a single-fiber (top row) and multiple-fiber (bottom row) voxel. Note the high resolution of the estimates (upper-right corner), where there is potential multiple orientations. In the clearly unidirectional voxel (bottom row), the smoothed ODF exhibits greater symmetry than the Funk transform on the original data (lower-left corner). No symmetry is enforced by the estimation procedure.*

References

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