

Quantitative Relations of Axial and Radial Diffusivities to Anisotropy Indices in Diffusion Tensor Imaging

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Introduction

Radial and axial diffusivities from Diffusion Tensor Imaging (DTI) measurements in diseases have been suggested to detect more specifically the axonal damage and demyelination respectively, in comparisons to other commonly used diffusion anisotropy indices (DAIs)¹. In this study the relations of sensitivities of radial and axial diffusivities with respect to other commonly used DAIs were evaluated. Monte Carlo Simulation² was performed to evaluate the deviation of the accuracy of these indices due to different signal to noise ratios (SNR). The correlations between diffusivities and DAIs were examined to facilitate the optimal selection of diffusion anisotropy measurement parameters.

Methods

Diffusion anisotropy indices are parameters indicating the trend of directional dependent MR signals. They can be classified into rotational variant indices (ADC, diffusivities), the rotational invariant indices in terms of eigenvalues (FA, RA, sRA, VR, VF, UA), and indices based on both eigenvalues and eigenvectors (LI, Add)³. In this study the trace of the tensor was constrained with a mean diffusivity $2 \times 10^{-3} \text{ mm}^2/\text{s}$. Diffusion ellipsoids were simulated from oblate, sphere to prolate shapes, and corresponding relations of nine DAIs with respect to the axial and radial diffusivities are plotted in Fig 1. The sensitivity of FA, RA and VF with respect to the axial and radial diffusivities is compared in Fig. 2. Furthermore, Monte Carlo simulations were performed to demonstrate the variation of diffusivities in the prolate ellipsoids (eigenvalues = $(1.5, 0.3, 0.3) \times 10^{-3} \text{ mm}^2/\text{s}$) corresponding to a rotation angle with respect to y, x, z axis $(0, 30^\circ, 15^\circ)$. The ratios of diffusivity were first assigned (5:1:1). For different diffusion directions (ellipsoid angles), the elements of diffusion tensors were calculated and the corresponding signals were derived. Gaussian noise was added with increasing SNRs to calculate the deviation of eigenvalues from their true values. The results are shown in Fig 3.

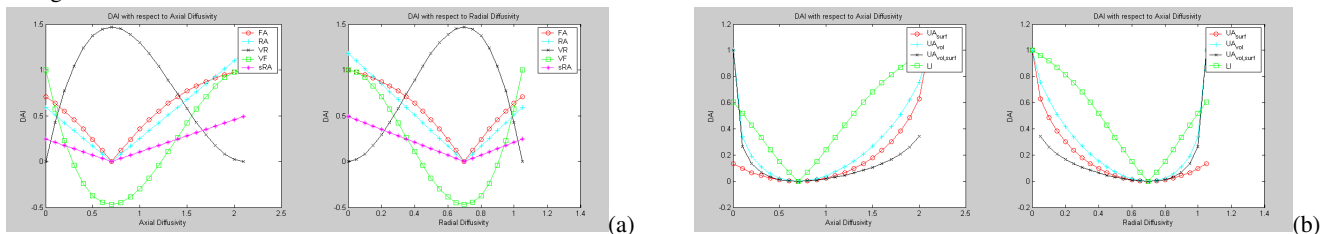


Fig 1. Relations of DAIs with respect to the Axial (a) and Radial (b) diffusivities (all the indices are in units of $10^{-3} \text{ mm}^2/\text{s}$).

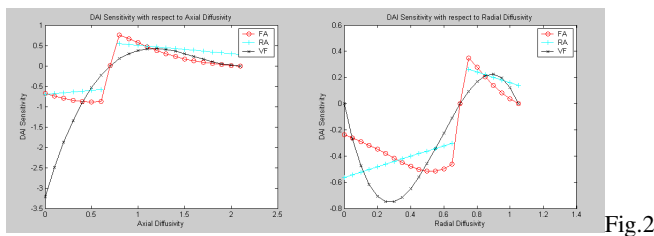


Fig 2. The sensitivity of FA, RA and VF with respect to the axial and radial diffusivities

Results and Discussion

From Fig 1, the value of FA/RA/sRA is the lowest for the sphere and the highest in prolate ellipsoids while VF/VR, for which the range exceeds one, is the same in extreme cases for both prolate and oblate. In the range of axial diffusivity from 0 to 1.75, $FA > RA > sRA$; in the range of radial diffusivity from 0.2 to 1.05, $FA > RA > sRA$. As axial diffusivity > 0.2 , $LI > UA_{vol} > UA_{vol,surf} > UA_{surf}$. As radial diffusivity < 0.95 , $LI > UA_{vol} > UA_{surf} > UA_{vol,surf}$. LI , UA_{vol} , UA_{surf} are larger in the extreme prolate cases. However, $UA_{vol,surf}$ is larger in the extreme oblate case. In the sensitivity derivation of differential of FA/RA/VR shown as Fig 2, for one unit change in radial diffusivity, the largest variation is 0.55 in RA, 0.5 in FA, and 0.75 in VF/ and 0.7 in RA, 0.8 in FA for axial diffusivity. The results indicate that all the variations are smaller than 1, which suggest that radial and axial diffusivities are more sensitive than other DAI indices. However, SNR can also influence the diagnosis accuracy. As shown in Fig.3, for a prolate case, the eigenvalues of diffusion tensor deviate from the true value increasingly as SNR decreases. FA, RA, UA_{surf} , $UA_{vol,surf}$ maps in a normal subject are shown in Fig 4. FA reveals higher SNR compared to the RA map. UA maps depict higher contrast than both FA and RA.

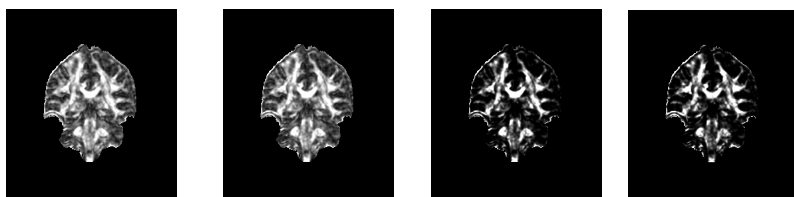


Fig 4. From left to right: FA, RA, UA_{surf} and $UA_{vol,surf}$ maps of a normal brain (GE Signa 1.5T, Single Shot pulsed-gradient SE-EPI, 6 directions, NEX=6, total scan time:9:36, TR/TE =8000/85ms, b=1000 s/mm², FOV = 24 cm)

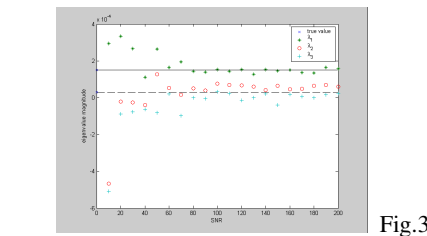


Fig 3. Monte Carlo simulations of SNR effects

References:

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3. Peter B. Kingsley, Concepts Magn Reson 2006;28A(2):123-154