Redundant Spatial Harmonic Information in Zeugmatography with Linear Encoding (R-SHIZLE) Theoretically Encodes Intra-Acquisition Decay

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Introduction: Much work has been done using time series of T_2* or T_2 weighted images to consider physiologic processes ([1], [2]). Such weighted images include contributions from other weighting factors including T_1, proton spin density, and inflow effects. Such contributions limit the ability to extract quantifiable measurements from a single image acquisition after a single excitation. To obtain absolute measurements, multiple image acquisition techniques have been used. Additionally, to calculate differential measurements, multiple acquisitions from separate excitations during different conditions have been used. Here we provide theoretical work for making quantitative measurements of T_2* or T_2 from a single image acquisition following a single excitation pulse by utilizing the expected symmetry of k-space observations.

Theory: The signal acquired in MRI can be modeled as the complex-valued Fourier transform of the object being imaged. As the object being imaged is physical, its ideal image representation is real-valued. A fundamental property of the Fourier transform is that conjugate symmetry exists about the origin in the k-space representation of a real-valued image. However, the k-space data is acquired over a period of discrete time points, with each k-space observation being acquired at a different time point and thus weighted by a different amount of intra-acquisition (T_2 or T_2*) decay. Additionally, unmodeled magnetic field inhomogeneity leads to a complex-valued image-space representation of the object being imaged. Both of these complications break the expected conjugate symmetry of k-space observations about the origin. The signal at a time point can be shown to be $S(k_x,k_y\,|\,t)=\iint \rho(x,y)\exp[-t/T_2(x,y)]\exp[i\gamma B(x,y)t]\exp[-i2\pi(k_x x+k_y y)]\;dxdy$. Acquisition of a B-field map by a chosen method ([3], [4]) and correction ([3], [4], [5]) can correct the non-symmetric effects of B-field inhomogeneities, thereby leaving the k-space non-symmetry caused only by intra-acquisition decay. Symmetric points can be considered in terms of the leading two terms of the Taylor expansions of the decay exponentials: $S(k_x,k_y\,|\,t_1)\approx \iint \rho(x,y)(1-t_1/T_2(x,y))\exp[-i2\pi(k_x x+k_y y)]\;dxdy$; $S(-k_x,-k_y\,|\,t_2)^*\approx \iint \rho(x,y)(1-t_2/T_2(x,y))\exp[-i2\pi(k_x x+k_y y)]\;dxdy$. The difference of these signals is: $S(-k_x,-k_y\,|\,t_2)^*-S(k_x,k_y\,|\,t_1)\approx(t_2-t_1)\iint \rho(x,y)/T_2(x,y)\exp[-i2\pi(k_x x+k_y y)]dxdy$. To first order, the intra-acquisition decay can then be estimated as:

$$T_2(x,y)\approx\frac{F^{-1}\left(S(k_x,k_y\,|\,t_1)\right)}{F^{-1}\left(\dfrac{S(-k_x,-k_y\,|\,t_2)^*-S(k_x,k_y\,|\,t_1)}{t_2-t_1}\right)}$$

Methods: Signal was simulated in MATLAB. The T_2 map was considered as a Shepp-Logan phantom scaled from 1e-6 to 1, the proton spin density map was unity in the regions of the T_2 map > 1e-6 and zero elsewhere, and a linear horizontal magnetic field gradient from -1e-6 T to 1e-6 T was considered. Each k-space observation point from a 96×96 acquisition was generated separately using the signal equation shown above, assuming that the central k-space point was acquired 65 ms after the excitation with a moving race track EPI readout [3] with an effective echo spacing of 0.768 ms and 64 race track lines. The magnetic field was corrected using the time-segmented method of Noll et al. [5]. The resulting corrected k-space data were used to compute an estimated T_2 map using the described method.

Results: The results of the simulation are shown in the accompanying figures. The simulated T_2 map (Fig. 1), proton spin density (Fig. 2), and corrupting B-field (Fig. 3) are shown in the first row. The image resulting from the simulated k-space data (Fig. 4) includes geometrical distortion from the magnetic field perturbation. Computation of the T_2 estimate leads to Fig. 5. The percentage error of the estimated T_2 map is generally below 10% (Fig. 6).

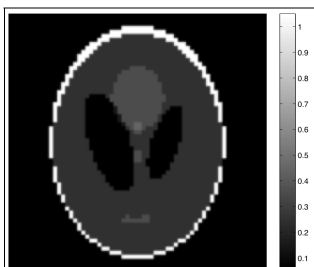
Fig. 1: Base T_2 Parameters

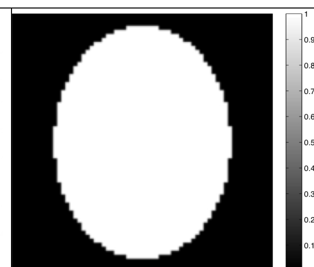
Fig. 2: Base Proton Spin Density

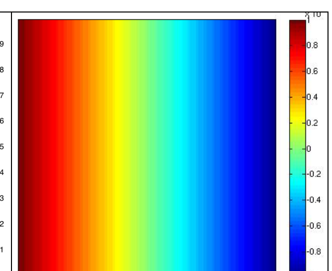
Fig. 3: Base B-field Perturbation

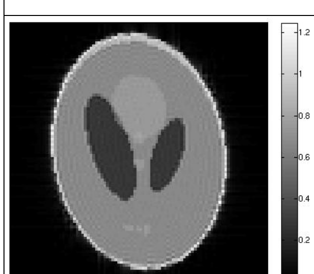
Fig. 4: Simulated Warped Image

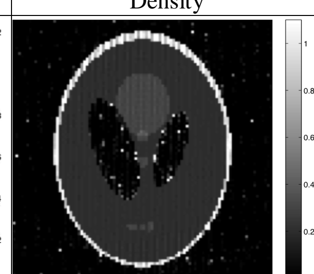
Fig. 5: Estimated T_2 Map

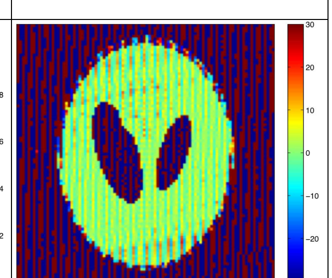
Fig. 6: Estimated T_2 Percent Error

Discussion: The simulation illustrates that the described method offers a possible means of estimating a T_2 map within a single EPI acquisition, even under the circumstance of non-negligible magnetic field inhomogeneity. The applicability of the two term Taylor expansion of the intra-acquisition decay exponential varies with the expected T_2 value, becoming suboptimal with shorter T_2 values. Nevertheless, with T_2 values on the order of tens of milliseconds, using the broken symmetry of an EPI acquisition should yield accurate results as illustrated in the above simulation. Multiple acquisitions of the center like of k-space in the form of navigator echoes are essential so that the center point may be sampled at separate times and the global T_2 estimated. Further implementational issues have yet to be addressed. The non-uniformity of the RF coil can lead to complex-valued images, thereby breaking the symmetry of k-space. It is expected that proper consideration of RF coil inhomogeneity coupled with dynamic magnetic field mapping will allow this method to be reliably implemented to consider dynamic changes in intra-acquisition decay in echo planar time series.

References: [1] Rosen, et al. MRM 14: 249-265. [2] Ogawa, et al. PNAS 89: 5951-5955. [3] Roopchansingh, et al. MRM 50: 839-843. [4] Jezzard, et al. MRM 34: 65-73. [5] Noll, et al. MRM 25: 319-333. **Support:** Funded in part by NIH grants MH019992, EB000215, AG020279.