SparseSENSE: Randomly-Sampled Parallel Imaging using Compressed Sensing

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INTRODUCTION:
Since the advent of compressed Sensing (CS) (1), much effort has been made to apply this new concept to various applications (2,3).The most desirable property of CS in MRI application is that it allows sampling of k-space well below Nyquist sampling rate, while still being able to reconstruct the image if certain conditions are satisfied. Recent work (4,5) applied CS to reduce scanning time in conventional Fourier imaging and demonstrated impressive results. In this abstract, we investigate the structure of parallel imaging encoding matrix, and apply CS to parallel imaging to achieve an even higher reduction in scanning time than what can be achieved by each individual method alone. Our experiments show the the combined method, named SparseSENSE, can achieve a reduction factor higher than the number of channels.

THEORY
In CS theory, a signal $\mathbf{x}$ with a sparse representation in the basis $\mathbf{\Psi}$, can be recovered from the compressive measurements $\mathbf{y} = \mathbf{\Phi x}$ [1], if the $\mathbf{\Phi}$ and $\mathbf{\Psi}$ are incoherent (6). In MRI, incoherence is satisfied when $\mathbf{\Psi}$ is the identity matrix (or the finer scales of a wavelet transform) and $\mathbf{\Phi}$ is a random subset of rows from the discrete Fourier matrix (6). However, this random sampling of k-space in all dimensions is generally impractical as the k-space trajectories have to be relatively smooth due to hardware and physical considerations (5). Instead, Lustig et al (4,5) designed practical incoherent sampling scheme for conventional Fourier imaging which randomly undersamples Cartesian grid along the phase-encoding directions only and fully samples the readouts. In parallel imaging, the measurement matrix (i.e. sensitivity encoding matrix) $\mathbf{\Phi}$ is equivalent to a block Toeplitz matrix multiplied with a discrete Fourier matrix, where the elements of Toeplitz matrix are the Fourier transform of coil sensitivities. We have shown that if there is randomness in the Toeplitz matrix, $\mathbf{\Phi}$ is incoherent with most sparse basis such as wavelet. The degree of randomness in the Toeplitz matrix affects the required number of samples. Although random sensitivity profiles is a better option, random Cartesian sampling along phase encoding direction is more practical and easier to implement without modification of hardware. We use the same sampling strategy as (5). Specifically, we choose samples randomly with sampling density scaling according to a power of distance from the origin, because most of the energy of images is concentrated close to the k-space origin. The image is reconstructed from the undersampled k-space data acquired from an array of coils by solving the nonlinear optimization problem: *minimize* $\|\mathbf{\Psi x}\|_1$ *subject to* $\|\mathbf{\Phi x} - \mathbf{y}\|_2 < \varepsilon$ [2], where $\mathbf{x}$ is the desired image, $\mathbf{y}$ is the downsampled *k*-space data, and $\mathbf{\Phi}$ is the sensitivity encoding matrix (7) and ε is a small constant to control the fidelity of the reconstructed result to the measured data.

METHOD AND RESULTS
Phantom data were collected on a Hitachi Airis Elite (Kashiwa, Chiba, Japan) 0.3T permanent magnet scanner with a four-channel head coil and a single slice spin echo sequence (TE/TR = 40/1000ms, 8.4KHZ bw, 256*256 matrix size, FOV = 220 mm^2). 52 out of 256 k-space lines in phase-encoding direction are randomly picked to simulate a reduction factor 5. The sampling pattern is shown in Fig 1(d). Due to the piecewise smooth feature of the phantom, total variation (TV) was used for the sparse representation. The minimization in Eq. [2] was solved using the lagged diffusivity fix-point numerical algorithm (8), implemented in MATLAB (Mathworks, MA). Figure 1 (a) shows the sum-of-square reconstruction from the fully sampled data as a reference for comparison, and (b) shows the reconstruction from the proposed SparseSENSE after 90 iterations, and (c) shows the result from SENSE (7). The SparseSENSE reconstruction shows only very few discernable artifacts compared to the reference.

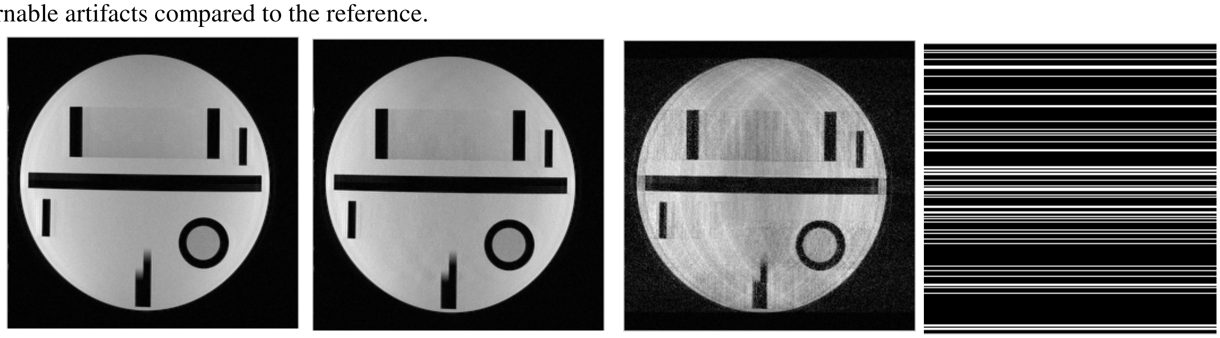

Fig.1 (a) Gold Standard (b) SparseSENSE (c) CG Method (d) Sampling Pattern

DISCUSSION AND CONCLUSION:
We apply CS to parallel MRI to achieve a reduction factor higher than the number of channels. The phantom experiments show promising results. The proposed reconstruction algorithm is computationally intensive, with a running time of 45 minutes for the phantom data on a 2.8GHz CPU/512MB RAM PC. Future work will test on real data with sparse representations such as angiography and investigate optimal sampling schemes.

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