Data reordering for improved constrained reconstruction from undersampled k-space data

G. Adluru[1], and E. V. DiBella[2]

[1]Electrical and Computer Engineering, University of Utah, Salt Lake City, Utah, United States, [2]Department of Radiology, University of Utah, Salt Lake City, U, United States

Introduction: There has always been interest in speeding the acquisition of MRI data by acquiring fewer data in k-space. Recently there has been a significant interest in applying inverse problem techniques to reconstructing images from undersampled k-space MRI data [1-6]. One class of methods, from the nascent field of compressed sensing, is based on the sparse representation of images. As an example of this approach, an image from incomplete k-space data is reconstructed by using a non-linear recovery scheme in which a spatial Total Variation (TV) constraint is applied on the estimated solution while preserving fidelity to the acquired data in k-space [1-3]. The method is best known to reconstruct piecewise constant data from its incomplete Fourier samples. Since most MR images are not exactly piecewise constant, the application of the method directly to undersampled MR image reconstruction is limited in terms of the quality of the reconstruction and accelerations that can be achieved. Here, we propose to use a data reordering technique to make the method more widely applicable to reconstructions in which the data do not perfectly match the constraints being used.

Methods: When full Fourier data of a signal are not available, reconstruction can be done by solving a convex optimization problem in which a cost function is minimized. The cost function consists of a fidelity term in which data fidelity is preserved at the locations where the data are sampled while minimizing a TV constraint. When the signal of interest is varying rapidly and is not smooth, the total variation of the signal is already high and hence reconstruction from incomplete Fourier data by minimizing the cost function is inaccurate. In the data reordering technique the undersampled data are reordered in the signal space according to an a priori determined ordering and then appropriate constraints are applied within an iterative reconstruction. Reordering the data can lead to lower total variation and sparser representations of the data in terms of finite differences and hence better reconstructions from the undersampled data can be obtained. Consider Fig 1 which shows a randomly varying signal and the corresponding curve sorted based on its intensities. The sorted curve is monotonic, smoothly varying and has lower TV as compared to the original curve. Hence reconstruction from incomplete k-space data is performed by minimizing the cost function *C* as shown below with reordering in the constraint term.

$$min(C) = \min_{\tilde{m}}\left[\left\|WF\tilde{m}-\tilde{d}\right\|_2^2 + \alpha\left\|\sqrt{\nabla\tilde{m}_r^{\,2}+\varepsilon}\right\|_1\right] \qquad \textbf{(1)}$$

In the above equation $\|.\|_2$ is the L_2 norm, $\|.\|_1$ is the L_1 norm, $\tilde{d}$ is the undersampled data acquired in *k* space, *W* is binary undersampling matrix, *F* is the Fourier transforms, $\tilde{m}$ is the estimated true signal, $\tilde{m}_r$ is reordered $\tilde{m}$, ∇ is gradient operator, ε is a small positive constant and α is the weighting factor for the constraint term. The method can be extended to 2D and multi-image data acquisition cases in MR by performing reordering in the constraint terms in the corresponding dimensions. Since the data we deal with in MRI is complex, reordering can be done independently for each pixel and the real and imaginary parts of the data.

Results: Fig 2 shows the results of reconstruction of the signal in Fig 1 from 50% of its full Fourier data. Fig 2a shows the original signal, the signal reconstructed from its undersampled Fourier data without reordering and the corresponding reconstructed signal with reordering. The sorting order of the full signal was used for reordering the undersampled signal. The reconstructed signal with reordering matches closely with the original signal. In practice it might not be possible to know beforehand the exact ordering as that obtained using full data. To simulate this case, the exact sorting order is randomly perturbed to see the effect of having inexact ordering on the performance of the algorithm. The degradation in the performance of the algorithm is more gradual rather than sudden when the accurate ordering is not known. Fig 2b illustrates the point. In the plot the x-axis represents the number of random perturbations i.e., number of indices of the exact 'sorting order vector' that are randomly perturbed. A value of 10 on the x-axis means that the values of the exact 'sorting order vector' at 10 distinct randomly picked indices (out of 70) are exchanged with those at a second set of 10 distinct randomly picked indices. The y-axis represents the log value of the total absolute difference between original full data and the reconstructed signal. We can see that as the number of random perturbations increases the total absolute error using reordering gradually increases, but is still better or comparable to that without the data reordering except for a few perturbations towards the end of the plot where the entire sorting order vector was randomly perturbed.

Fig 3 shows an example of a multi-image data reconstruction with reordering in space (*x* and *y* dimensions) and multi-image dimensions. Fourier data was generated from the magnitude DTI image data of a brain slice with different diffusion encoding directions (data provided by A. Alexander, U. Wisc.). Fig 3a shows the image of a single diffusion encoding direction reconstructed from full Fourier data using Inverse Fourier Transform (IFT). Fig 3b shows the reconstructed image from R~3 Fourier data (undersampled in variable density (VD) random fashion) with TV constraints in multi-image and space dimensions but with no reordering and Fig 3c shows the corresponding reconstructed image with reordering. Reordering in this case was done in two steps. In the first step the central low resolution data from VD undersampling was used for reordering the data in multi-image dimension only and reconstruction was performed. The resulting data were then used to determine the spatial ordering and final reconstruction was performed with reordering in the spatial and multi-image dimensions. We can see that Fig 3c matches Fig 3a better especially around the ventricles.

Discussion and Conclusion: Methods for obtaining optimal ordering of the data depend on the application, for example the central low resolution data that can obtained from variable density undersampling can be used to reorder the data in the multi-image dimension. In conclusion using data reordering in the constraint terms can improve reconstruction from undersampled data when the constraints do not directly match the data to be reconstructed.

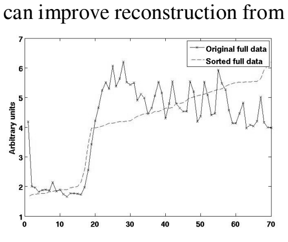
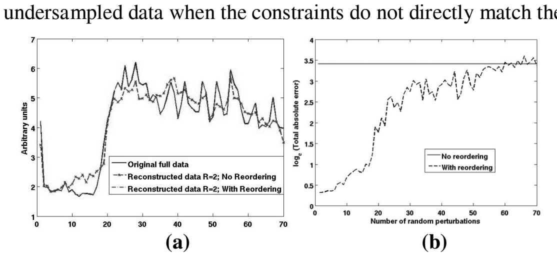
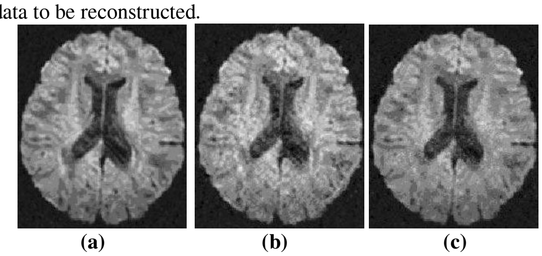


Fig 1. A randomly varying signal and its corresponding sorted signal.



(a) (b)

Fig 2. **(a)** Original signal and the reconstructed signals from R~2 Fourier data without and with reordering. **(b)** Plot comparing reconstruction errors without reordering and with reordering as a function of inaccuracies in exact ordering.



(a) (b) (c)

Fig 3. **(a)** Image of a diffusion encoding direction reconstructed using IFT from full Fourier data. **(b)** Image reconstructed from R~3 Fourier data without reordering. **(c)** Corresponding image reconstructed from R~3 Fourier data with reordering.

References: [1] Candes EJ et al, IEEE Trans Info Theory, 52: 489-509, 2006. [2] Block KT et al, MRM, 57: 1086-1098, 2007. [3] Lustig M et al, Proc. ISMRM, 685, 2005. [4] Portniaguine O et al, Proc. ISMRM, 481, 2003. [5] Adluru G et al, MRM, 57: 1027-1036, 2007. [6] Adluru G et al, Proc. IEEE ISBI, 109-112, 2007.