Prediction of HYPR and HYPR LR performance based on image sparsity and temporal correlation

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INTRODUCTION

HighlY constrained backPRojection (HYPR) [1] is an approximate reconstruction technique from severely undersampled data. Initial investigations demonstrate that HYPR can be successfully used in many time-resolved imaging applications. However, if the time series contains spatially distinct objects with significantly different time courses, then, in order to avoid cross-talk between these objects, the length of the temporal window for the composite images used in HYPR reconstruction has to be limited. It has long been intuitively understood that the reconstruction error in HYPR depends on the level of image sparsity as well as the degree of spatio-temporal correlation of the image series. The recently introduced Local Reconstruction HYPR (HYPR LR) [2] technique modifies the original HYPR algorithm in order to reduce the cross-talk between different objects in the image series and to allow for the use of longer temporal windows for the composite images that constrain the reconstruction.

It would be beneficial to be able to predict the performance of HYPR and HYPR LR, admissible undersampling factors and temporal windows for composite images in each application based on the a priori knowledge about signal configuration in the image series. In this work, we derive analytical estimates of the error of both HYPR and HYPR LR reconstructions and characterize them in terms of the spatio-temporal correlation of the time series and image sparsity.

THEORY AND METHODS

In the HYPR-based algorithms, the individual time frames are obtained by multiplying a composite image by a weighting image which is specific to each time frame. Composite images are obtained from all or a subset of projections acquired in the exam and determine spatial resolution and SNR of the reconstructed time frames, while the weighting images reflect the temporal behavior. The main difference between the HYPR and HYPR LR algorithms is in the formation of the weighting images. In HYPR, they are obtained by applying unfiltered backprojection to normalized Radon projection data. In HYPR LR, the weighting images are produced by taking the ratio of low pass filtered versions of the time frame and composite images. The filtering is performed via convolution, typically, with a Gaussian filter φ. .

We assume the multiplicative model of the time series behavior, that is, $T(\vec{x};t) = f(t)(1+\Delta f(\vec{x};t))C(\vec{x})$, where C is the composite image, $f(t)$ is the common-mode temporal behavior of the time series, and $\Delta f(\vec{x};t)$ is the deviation from the common-mode temporal behavior. Then the reconstruction error of HYPR and HYPR LR, respectively, can be estimated as

$$\left|H(\vec{x};t) - T(\vec{x};t)\right| \le 2\left|f(t)\right| \max_{\vec{y}\in \operatorname{supp} C}\left|\Delta f(\vec{y};t)\right| HSI(\vec{x}),$$

$$\left|H_{LR}(\vec{x};t) - T(\vec{x};t)\right| \le \left|f(t)C(\vec{x})\right| \max_{\vec{y}\in \operatorname{supp} \varphi}\left|\Delta f(\vec{x}) - \Delta f(\vec{x}-\vec{y})\right|.$$

Here, *HSI* is HYPR-specific sparsity index that is a new notion specific to the original HYPR algorithm. *HSI* measures the average deviation of the normalized composite image from a delta function along each image profile. The value of *HSI* depends only on the signal distribution in the composite image volume and undersampling factor of the acquisition. Note that the error of HYPR reconstruction depends on the maximal deviation from the common-mode temporal behavior Δf measured over the whole image volume, while for HYPR LR the deviation is measured only within the region of spatial averaging performed during the convolution process. This explains why images reconstructed with the HYPR LR algorithm exhibit significantly less of the cross-talk between spatially distinct objects with different time courses.

In order to determine a general trend, we consider a set of simulated signal configurations with variable degrees of temporal correlation and use them to predict acceleration factors providing acceptable reconstruction error with both HYPR and HYPR LR algorithms.

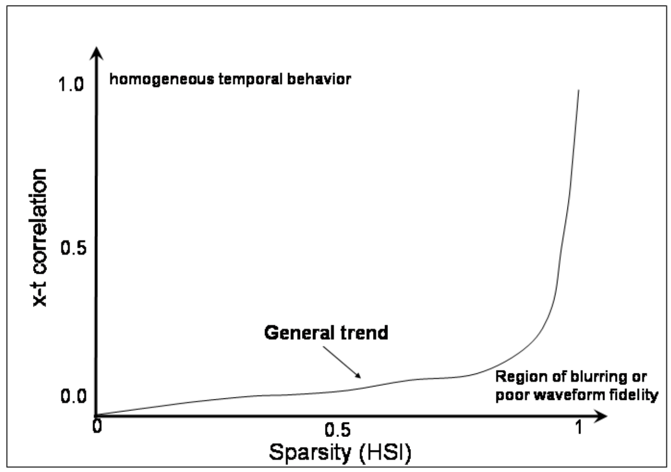

Figure 1. Dependence of the HYPR reconstruction error on spatio-temporal correlation and sparsity of the images in a time series. General trend curve corresponds to a 10% relative error.

RESULTS AND DISCUSSION

Error estimate for HYPR algorithm shows that HYPR can achieve perfect reconstruction if the time series exhibits homogeneous temporal behavior (all spatial locations in the image have the same time courses, $\Delta f = 0$) or the images are ideally sparse (*HSI* = 0). In more realistic cases, if one of these quantities is sufficiently small, HYPR will provide an accurate reconstruction. Unlike HYPR, HYPR LR performs well even if there is no global sparsity but temporal behavior in each local neighborhood (the size of which is determined by the size of the convolution kernel used in HYPR LR filtering) is relatively homogeneous.

The plot in Figure 1 illustrates a general trend for achieving a 10% relative reconstruction error using the original HYPR algorithm as a function of the spatio-temporal correlation and *HSI*. A similar plot was obtained for the HYPR LR algorithm based on the same set of simulated signal configurations and temporal behaviors. Then, to provide at least a qualitative indication of whether such plots can predict achievable acceleration for a contemplated clinical application we compared the obtained results with the coordinates and acceleration factors that were judged to be acceptable in the several applications we have begun to investigate. As expected, HYPR LR provided better reconstruction in the cases where the images were not globally sparse but exhibited high levels of spatio-temporal correlation locally.

CONCLUSIONS

Results obtained in a series of experiments demonstrate that spatio-temporal correlation values and sparsity index are indeed good predictors of the overall performance of HYPR and HYPR LR. It must be noted that iterative HYPR-based methods, such as CG HYPR or I-HYPR, require a separate investigation concerning the parameters that determine their reconstruction error.

REFERENCES

1. Mistretta CA, *et al.*, MRM, 55, 30, 2006. **2.** Johnson KM, *et al.*, MRM, in press.