L-map: Exploiting Spatial-Temporal Correlation of Phase in MRI

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INTRODUCTION

In many MR applications, it is always desirable to achieve high spatial and temporal resolution. However, simultaneous achievement of both goals is limited by the Nyquist theorem, hardware, safety limitations, and signal-to-noise (SNR). Undersampled radial acquisitions in conjunction with Highly Constrained backPRojection (HYPR) [1] and HYPR with Local Reconstruction (HYPR-LR) [2] have been developed to increase temporal resolution within a given scan time while maintaining high SNR and a low artifact level in magnitude data. However, in applications where phase information has to be preserved, such as Phase Contrast MR Angiography and Chemical Shift Imaging (CSI), special treatment of the phase images is required for HYPR or HYPR-LR processing [3]. Here we propose a novel method that incorporates the concept of generating a vector map or "L-map" that is capable of processing the phase images without using any information from the magnitude images. A numerical phantom simulation using HYPR-LR was implemented to illustrate the potential of this method.

METHODS

The original HYPR and HYPR-LR algorithms cannot be applied directly to phase sensitive image reconstruction due to phase wrapping. For example, blurring the phase images directly over the region where phase wrapping exists will produce phase errors along the wrapping line, as illustrated in Fig. 1. This problem can be solved by introducing an L-map, where L is defined as the magnitude of the complex difference between a unit reference vector and a phase vector $e^{i\theta} = \vec{s}/|\vec{s}|$, and $\vec{s}$ is the complex pixel value, as shown in Fig. 2(a). Mathematically, L is given by

$$L = |\exp(i\cdot\theta) - reference\_vector|$$

For simplicity, the reference vector can be chosen as $r_1 = \exp(i\cdot 0) = 1$. Because L is always positive and does not suffer from wrapping (zero and 2π phase result in the same L), HYPR and HYPR-LR techniques can be well implemented on L-map images producing a series of HYPR L-map images. After that, HYPR L-map images are inverse transformed back to phase angles using the law of cosines, which produces two possible angles for each L, i.e.

$$\beta = \pm\arccos\left(\frac{1^2+1^2-L^2}{2\cdot 1\cdot 1}\right) = \pm\arccos\left(1-\frac{L^2}{2}\right),$$

where β is the angular difference between the phase vector and the reference vector. The ambiguity of the sign can be removed by using another reference vector $r_2 = \exp(i\cdot\pi/2) = i$ and calculating the corresponding HYPR L_2 images and β_2 (Fig. 2(b)). Based on the knowledge of L_1 and L_2 the final phase angle θ can be correctly determined (Fig. 2(c) caption).

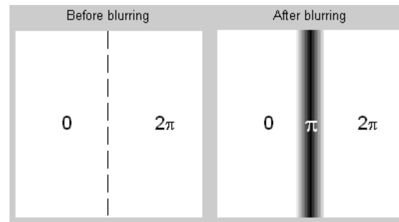

Figure 1. Before blurring, both zero and 2π phase give a positive real channel signal. However, blurring process produces a band of π phase, which means negative real channel signal.

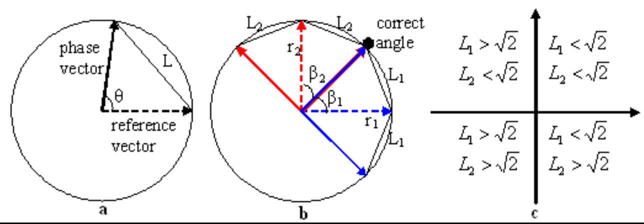

***Figure 2.** **(a)** Definition of L-map. **(b)** The ± ambiguity can be resolved by introducing L_2. One solution from L_1 (blue) and one from L_2 (red) will overlap producing the correct result. **(c)** A general scheme of calculating the correct angle. For example, $L_1 < \sqrt{2}$ and $L_2 < \sqrt{2}$ means that phase vector is in the first quadrant of the complex plane and θ can be calculated as $\theta = (|\beta_1| + \pi/2 - |\beta_2|)/2$.*

A simulation of L-map HYPR-LR method for phase reconstruction was implemented in Mablab (Mathworks, Natick, MA). A series of input images were generated simulating fat and water. Each time frame contained only 15 projections and was sampled at an echo time of 12 us and incremented by 80 us for the next time frame. This corresponds to an acceleration factor of 17 compared to Cartesian sampling. All projections were interleaved into 54 groups, with each group covering the whole k-space sparsely and uniformly. Image size was 256×256 and Gaussian white noise with zero mean and a variance of 10% of the signal amplitude (SNR=10) was added to both real and imaginary channels separately. The proposed L-map HYPR-LR method was performed in three steps: 1. Transform the phase of the Filtered Back-Projection (FBP) images into L_1 and L_2 map images. 2. Perform HYPR-LR reconstruction of these two sets of L-map images, generating HYPR L-map images. 3. Use the scheme in Fig. 2(c) to calculate the correct phase. The HYPR-LR parameters were: sliding composite window length = 15 frames, Gaussian blurring kernel with σ = 7 pixels and filter size = 10 pixels.

RESULTS

Fully sampled, undersampled and L-map HYPR-LR reconstructed phase images are shown in Fig. 3 (a,b,c). It is apparent that noise is significantly reduced in regions simulating water (inner disk) and fat (outer ring) after L-map HYPR-LR reconstruction. L-map is also used to compare the reconstructed phase to the actual phase, as shown in Fig. 3(d). For fat and water, the reconstructed phase is in good agreement with the actual phase.

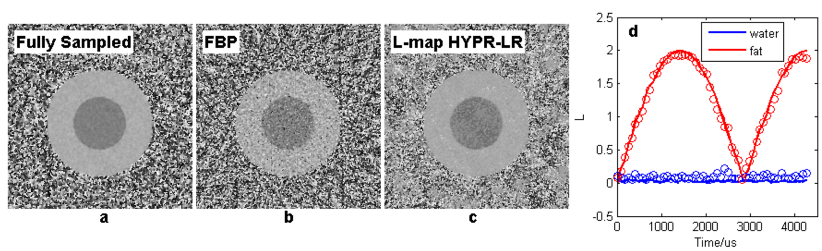

*Figure 3. Fully sampled phase image **(a)** is compared with FBP reconstruction **(b)** and L-map HYPR-LR method **(c)**. The L values corresponding to the actual phase (solid line) and to the L-map HYPR-LR reconstructed phase (open circle) from regions of interest placed in the water (blue) and fat (red) are compared **(d)**.*

DISCUSSION AND CONCLUSIONS

This simulation demonstrates that by introducing the L-maps, phase sensitive images can be properly reconstructed using the HYPR-LR algorithm. The L-map accurately determines phase without wrapping and avoids potential signal cancellation in the formation of the HYPR-LR composite images [4]. Future work includes using the magnitude image to suppress background random phase prior to HYPR-LR processing and extending the L-map concept to other reconstruction techniques for phase processing.

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