Regularized B1+ Map Estimation with Slice Selection Effects

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Introduction

In MRI, a map of the B_1^+ field strength, called a B_1^+ map, is essential in many situations. For parallel transmit excitation (using a coil array), *e.g.,* [1], one must have a map of the B_1^+ magnitude and phase for RF pulse design. At high fields (≥ 3T) B_1^+ inhomogeneity creates spatially varying signal and contrast; a B_1^+ map allows for proper pre-scan calibration [2]. A conventional approach to B_1^+ mapping is to collect two scans, one of which uses twice the RF amplitude of the other [3]. Using the double angle formula, a standard method-of-moments estimator is used that ignores noise in the data. This estimator performs poorly in image regions with low spin density. This simple approach also does not allow for more than two angles nor does it account for more complicated physical factors such as slice selection effects. Slice selection perturbs the linear relationship here [4] and causes absolute and distribution error in the flip angle [5]. We propose a new approach that incorporates multiple coils and multiple tip angles as well as accounts for noise in the model. We extend our previous work [6] to incorporate a known slice profile to account for slice selection effects. The proposed iterative regularized estimator estimates both the unknown B_1^+ magnitude and phase map from multiple reconstructed images.

Theory

Using K transmit coils and a common receive coil and a sequence of L nominal tip angles with known relative RF amplitudes a_l gives KxL reconstructed images that we model as follows:

$$y_{jkl} = f_j e^{i\varphi_{jk}} F_l(x_{jk}) + \varepsilon_{jkl} \quad \text{where} \quad F_l(x_{jk}) = \int \sin(a_l x_{jk} \sigma(z))dz \qquad l=1 \ldots L,\ \ j=1,\ldots N,\ \ k=1,\ldots,K \tag{1}$$

where f_j is the underlying object magnetization in the jth voxel, φ_{jk} is the phase of the kth coil at the jth voxel, x_{jk} is the unkown "B_1^+ map" and ε_{jkl} denotes the zero-mean complex Gaussian noise. Here, F_l takes the place of the sin seen in the double angle formula and is based on the MR signal equation for slice selection, where σ(z) is the achieved slice profile. In the case of a perfect slice selection profile (*i.e.*, σ(z) = rect(z)), the model simplifies to that in [6]. Using the model in Eq. 1, we jointly estimate the B_1^+ magnitude maps x and the phase maps φ by finding the minimizers of the following penalized-likelihood cost function:

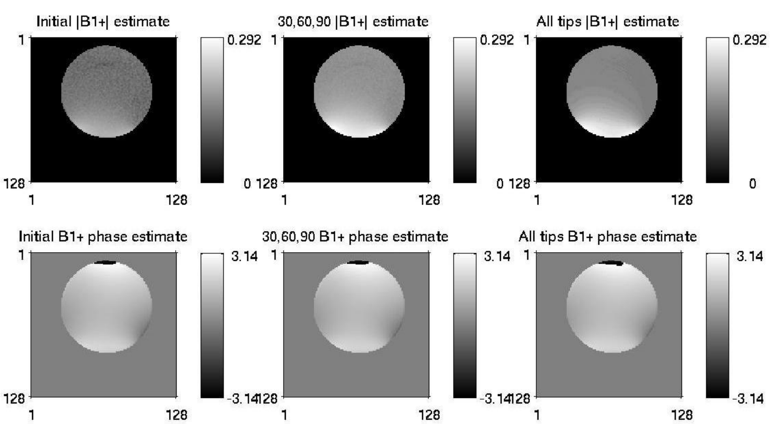
$$\Psi(x,\varphi,f) = \sum_{k=1}^{K}\sum_{j=1}^{N}\sum_{l=1}^{L} \frac{1}{2}\left| y_{jkl} - f_j e^{i\varphi_{jk}} F_l(x_{jk})\right|^2 + \beta_1 R(x_k) + \beta_2 R(\varphi_k) \tag{2}$$

where $R(x_k)$ *and* $R(\varphi_k)$ are regularizing roughness penalties where β_1 and β_2 control the smoothness of the estimates. Because there is no analytical solution for the minimizer over all three sets of parameters, we use a block alternating minimization approach: cycling over each of the parameter types and minimizing with respect to one parameter while holding the other two constant at their most recent values. The minimizer with respect to f can be found analytically; the minimizers for the phase φ and magnitude map are found using principles of optimization transfer, leading to the following update:

$$f_j = real\left(\frac{\sum_{k=1}^{K}\sum_{l=1}^{L} e^{i\varphi_{jk}} F_l(x_{jk}) y_{jkl}}{\sum_{k=1}^{K}\sum_{l=1}^{L} \left|F_l(x_{jk})\right|^2}\right) \qquad x_k^{(n+1)} = x_k^{(n)} - diag\left\{\frac{1}{\sum_{l=1}^{L} a_l^2 \left|f_j\right|^2 \left(\int \sigma(z)dz\right)^2 + r\beta_1}\right\}\nabla_{x_k}\Psi(x_k^{(n)},\varphi,f)$$

$$\varphi_k^{(n+1)} = \varphi_k^{(n)} - diag\left\{\frac{1}{\sum_{l=1}^{L}\left|y_{jkl} f_j F_l(x_{jk})\right| + r\beta_2}\right\}\nabla_{\varphi_k}\Psi(x\ \ ,\varphi_k^{(n)},f) \qquad \varphi_{jk}^{(0)} = \angle\left(f_j \sum_{l=1}^{L} F_l(x_{jk}) y_{jkl}\right) \tag{3}$$

The factor "r" depends on the choice of the regularizer. The iteration is initialized using the standard double angle estimate for x, the computational formula for f and the phase as shown in Eq. 3. β_1 and β_2 are chosen based on spatial resolution analysis to achieve a desired FWHM [7].



Experiments

We imaged a phantom with a surface transmit coil positioned to create a B_1^+ map that was larger on one side. We obtained images at eighteen tip angles from 10° to 180° using a TR of 8 seconds We used an assumed slice profile of a gaussian. We estimated the B_1^+ map based on all eighteen tip angles as well as with just three of the tip angles (30, 60 and 90) using 200 iterations of the algorithm. The regularized estimates are much smoother than the conventional estimate. If we consider the estimate using all the data as the "truth", the conventional estimate has a masked NRMSE of 19.7% compared to only 7.8% for the regularized estimate using 3 tip angles.

Discussion and Conclusion

We have described a new regularized method for B_1^+ that estimates both the B_1^+ magnitude and phase. This method allows for multiple coils, arbitrary selection of angles, and incorporates slice selection effects. This method yields B_1^+ maps that interpolate smoothly over regions with low spin density. Experimental results show a decrease in the NRMSE using only three angles over the conventional method. This method also estimated the B_1^+ phase which can be used for designing pulse sequences in parallel excitation. This new method gives increased accuracy, especially at higher tip angles where slice selection effects are most important.

References [1] Katscher *et al*, MRM, 49:144-150, 2003. [2] Cunningham *et al*, MRM, 55:1326-1333, 2006. [3] Akoka *et al*, Mag. Res. Im, 11:437-441, 1993. [4] Stollberger *et al*, MRM, 35:246-251, 1996. [5] Wang *et al*, MRM, 56:463-468, 2006. [6] Funai *et al*, Proc. IEEE ISBI 2007, 616-619. [7] Fessler *et al*, IEEE Trans. Im. Proc., 5:1346-1358, 1996.

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