Optimal Phase-Relaxed Design of Small-Tip-Angle Parallel Transmission RF Pulses

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INTRODUCTION

In conventional small-tip-angle (STA) parallel transmission pulse design, RF pulses are designed to drive transverse magnetization to match a target magnetization with a desired magnitude profile and a flat phase profile [1-3]. This is overly restrictive because a) the phase profile is not of interest in most of the MRI applications and b) parallel transmit coils do not necessarily favor excitation with a flat phase. Kerr et al. [4] proposed the phase-relaxed parallel transmission pulse design, where a spatially varying target phase is predetermined to improve the performance of the pulse. Instead of choosing a predetermined target phase in the conventional design formulation, we explicitly remove the phase constraint and reformulate the pulse design problem as an optimization problem with non-quadratic cost function. We show that the gradient vector of the cost function can be expressed as a closed form, based on which a nonlinear CG algorithm is used to efficiently solve the optimization problem. Due to the convexity of the cost function, the globally minimum error of the magnitude profile (in combination with an RF power regularization term) is achieved. The optimal phase profile that gives rise to the minimum magnitude error is also found as a byproduct of the proposed method.

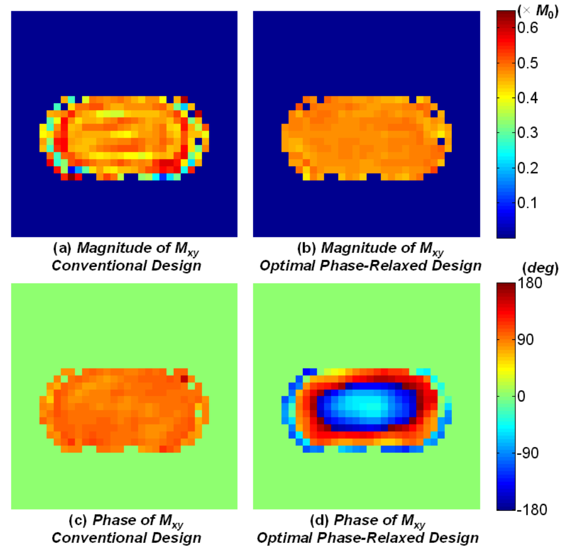

Fig. 1. Bloch simulation results comparing M_{xy} of two 30º RF excitation pulses for B_1 inhomogeneity correction. The RF pulse based on the conventional design creates significant uncorrected inhomogeneity in the magnitude profile (a) due to the overly restrictive flat phase constraint (c) (90º in this case). The RF pulse based on the optimal phase-relaxed design achieves a much more homogenous magnitude (b) because it finds the optimal phase (d) that minimizes the magnitude error. The RF power is also reduced from 13.1 to 2.9 (a.u.) with the optimal phase-relaxed design.

PROPOSED METHOD

Problem Formulation. The optimized phase-relaxed parallel transmit pulse design is formulated as:

$$\text{Choose } \mathbf{b} \text{ to minimize } J(\mathbf{b}) = \||\mathbf{Sb}|-\mathbf{p}\|_{\mathbf{W}}^2 + \lambda\|\mathbf{b}\|_2^2, \qquad (1)$$

where $\mathbf{p} = [p_1, p_2, \ldots p_M]^T$ is a length-M vector of the desired magnitude profile samples, $\mathbf{b}$ vertically concatenates $\mathbf{b}_l$ (a length-N vector of the RF pulse samples of the lth coil), $\mathbf{S}$ horizontally concatenates $\mathbf{S}_l$ (an $M \times N$ matrix whose (u,v)-th element is $(\mathbf{S}_l)_{u,v} = i\gamma\Delta t s_l(\mathbf{r}_u) e^{i\mathbf{k}((v-1)\Delta t)\cdot\mathbf{r}_u}$ where Δt is the temporal step size, $s_l(\mathbf{r})$ is the transmit sensitivity of the lth coil, and $\mathbf{r}_u$ is the space location), $l = 1,2,\ldots,L$, $|\mathbf{Sb}|$ denotes the absolute value of $\mathbf{Sb}$, the weighted l_2 norm $\||\mathbf{Sb}|-\mathbf{p}\|_{\mathbf{W}}^2 = (|\mathbf{Sb}|-\mathbf{p})^T\mathbf{W}(|\mathbf{Sb}|-\mathbf{p})$ for a given spatial weighting matrix $\mathbf{W} = \text{diag}\{w_1, w_2, \ldots, w_M\}$ represents the magnitude error of the magnetization profile, the l_2 norm term approximates RF power, and λ is a regularization parameter.

***Gradient Vector of J*(b)**. Eq. (1) can be rewritten as a sum of weighted l_2 norms and an l_1 norm term (details omitted due to limited space). The complex gradient of $J(\mathbf{b})$ can then be expressed as:

$$\nabla_{\mathbf{b}} J(\mathbf{b}) = 2(\mathbf{S}^H\mathbf{US}+\lambda\mathbf{I})\mathbf{b}, \quad \text{where } \mathbf{U} = \text{diag}\left\{w_m - p_m^2 w_m^2 \Big/ \sqrt{p_m^2 w_m^2 |(\mathbf{Sb})_m|^2 + \delta}\right\}_{m=1,\ldots,M}, \qquad (2)$$

$\mathbf{S}^H$ represents the Hermitian transpose of $\mathbf{S}$, $|(\mathbf{Sb})_m|$ denotes the absolute value of the mth component of the length-M vector $\mathbf{Sb}$, and δ is a small constant introduced to overcome the nondifferentiability of l_1 norm at the origin [5].

Optimization Algorithm. We adapt a standard nonlinear CG algorithm [5] to the complex parameter case to numerically solve the optimization problem in Eq. (1). The algorithm starts with the conventional STA RF pulses (with the flat phase constraint) [1-3] and then improves the solution over the CG iterations. In each iteration, a backtracking line search is applied to find the optimal step size [6] and Eq. (2) is used to compute the gradient vector. Due to the convexity of $J(\mathbf{b})$, the algorithm converges to the globally optimal $\mathbf{b}$. As a byproduct, the optimal phase that minimizes the combined magnitude error and RF power can also be obtained. With MATLAB implementation running on a 2.4 GHz workstation with 8Gb RAM, it takes about 20 sec to 2 min (depending on matrix size) for the algorithm to converge.

RESULTS

B_1 Inhomogeneity Correction. The first result is based on simulation of dual-channel transmission of 30° RF excitation pulses for B_1 inhomogeneity correction. The transmit sensitivity maps are acquired using a dual-channel transmission 3T GE Signa scanner with a torso phantom. FOV is 48 × 48 cm^2 and the matrix size is 32 × 32. A 8-turn unaccelerated inward spiral is used to cover the excitation k-space. Pulse length = 1.8 msec for both the STA and optimal phase-relaxed designs. As shown in Fig. 1, the magnitude of M_{xy} based on the optimal phase-relaxed design is much more homogenous than that based on the STA design (standard deviation = 0.014 vs. 0.08). This is because the STA design uses an overly restrictive flat phase (90º in this case) while the optimal phase-relaxed design chooses the optimal phase to minimize the magnitude error. The RF power is also significantly reduced from 13.1 to 2.9 (arbitrary units) by using the optimal phase-relaxed design.

Reduced FOV Excitation. The second result is based on simulation of eight-channel transmission pulses for reduced FOV excitation. The desired magnitude profile is sin15º inside a centered infinite cylinder (diameter = 12 cm) and zero outside (FOV = 32 × 32 cm^2). The transmit sensitivities are created by FDTD software to simulate a transmit array at 7 T. The number of spiral turns = 12/R and pulse length = 4/R msec, where R is the reduction factor and R = 1,2,3,4,6,12. In Fig. 2, the optimal phase-relaxed design shows improvement over the STA design at each R with respect to a) standard deviation of the passband signal (signal inside the cylinder), b) standard deviation of the stopband signal (signal outside the cylinder), and c) RF power. The improvement becomes increasingly significant as R increases because the number of time points becomes increasingly insufficient to produce good spatial selectivity without excessive power, and relaxing the phase creates additional degrees of freedom that are able to improve selectivity in the magnitude profile and/or reduce RF power.

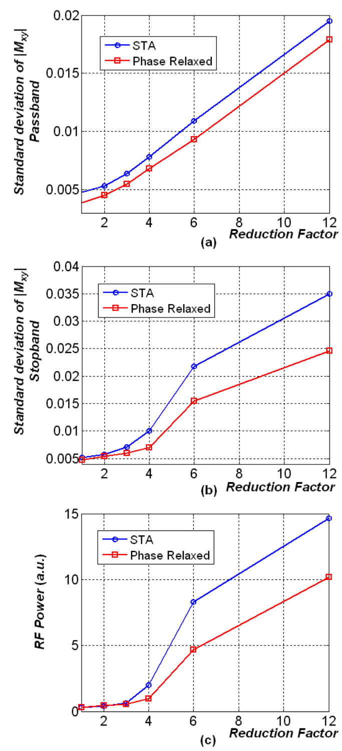

Fig. 2. Bloch simulation results comparing RF pulses designed by STA and the optimal phase-relaxed methods at various reduction factors.to excite an infinite cylinder in a square FOV. (a) Standard deviation of $|M_{xy}|$ inside the cylinder (passband), (b) standard deviation outside (stopband) and (c) RF power.

CONCLUSION

The optimal phase-relaxed design removes the phase constraint in the conventional STA design, which essentially optimizes the magnitude profile of the final magnetization over all possible phase profiles. Bloch simulation results demonstrate that the optimal phase-relaxed design can achieve significantly better magnitude profile and/or lower RF power than the conventional STA design for both the B_1 inhomogeneity correction and reduced FOV excitation applications.

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