Practical Optimum Experimental Designs for Fast T1 Relaxometry with SPGR Sequences

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Introduction: Knowledge of the longitudinal relaxation time T_1 is necessary in many quantitative MRI applications. T_1 mapping using variable flip angle SPGR acquisitions is an attractive choice due to its speed and accuracy [1]. Critical to the efficiency of the method is the choice of flip angles. A pair of optimized angles, *ideal angles*, may be easily computed for a single T_1 value [1]. However, more angles may be necessary for optimized performance over a range of T_1 values observed in both healthy and pathological tissues and different tissue types. A weighted genetic algorithm (wGA) was used to optimize estimation accuracy for a single T_1 value and yielded a 10 flip angle design with improved performance over a wider T_1 range [2]. In this work, we describe a method for automatic selection of T_1 mapping flip angles, which explicitly optimizes the performance of T_1 mapping for a wide range of T_1 values and demonstrates performance similar to the 10 flip angle wGA design with as few as 3 excitation flip angles, which may allow development of more flexible and efficient T_1 mapping protocols.

Theory: The SPGR signal equation is given by $S = M_0(1-E_1)\sin\alpha / (1-E_1\cos\alpha)$, where M_0 is equilibrium longitudinal magnetization, α - excitation flip angle, T_R - repetition time, and $E_1 = e^{-T_R/T_1}$. The lowest limit of the variance of T_1 estimate $\sigma^2(\xi, T_1)$ is equal to $\left[\mathbf{M}^{-1}\right]_{2,2}$, where **M** is the Fisher information matrix with elements $[\mathbf{M}]_{kl} = \sum_{i=1}^{n}\left(w_i \frac{\partial S(\xi_i,\mathbf{p})}{\partial p_k}\frac{\partial S(\xi_i,\mathbf{p})}{\partial p_l}\right)$ [3]. Here, $\mathbf{p} = [M_0\ T_1]$, $k,l = 1..2$, and ξ_i is a vector of control variables for the i^{th} measurement ($\xi_i = \alpha_i$). The weight w_i indicates the relative accuracy of the i^{th} measurement that is acquired, and is equal to the number of time averages for a particular measurement. We chose to optimize T_1 mapping efficiency given by $\Gamma(T_1) = T_1 / \left(\sigma(\xi, T_1)\sqrt{t}\right)$, where t is total acquisition time. It is desirable to get maximized and uniform efficiency in the expected T_1 range given by $T_1^{(j)}$, j=1…m. We propose several optimization criteria to fulfill this goal - minimum efficiency $\left(C_1 = \min\left[\Gamma\left(\xi, T_1^{(j)}\right)\right]\right)$, algebraic mean $\left(C_2 = \sum_{j=1}^{m}\Gamma\left(\xi, T_1^{(j)}\right)\right)$, and geometric mean $\left(C_3 = \left[\sum_{j=1}^{m}\Gamma^{-1}\left(\xi, T_1^{(j)}\right)\right]^{-1}\right)$ in the range.

Methods and Results: In our simulations, the criteria were maximized using simulated annealing optimization to avoid local minima [4]. The flip angles were sought in the range [1°, 60°]. The optimization range was [0.1, 5] s. We compared the new methods against the wGA algorithm, (FA=(2,3,4,5,7,9,11,14,17,22), T_R=5 ms), the heuristic algorithm in [5], and the variance minimization approach of [6]. All three criteria improved mean efficiency gain compared to the wGA algorithm using just 3 flip angles (Fig. 1). Criterion C_2 yielded a performance curve similar to the wGA design. C_2 and C_3 are best at maximizing efficiency, while C_1 and C_3 provide more uniform response in the range (Fig. 1). It is clear from Table 1 that increasing the number of flip angles from 3 to 10 results in an average gain of only 5-7% in maximum values of the optimization criteria. Furthermore, flip angles of the 10 point design are clustered around 3 values (for Criteria 1 and 2). Hence, we suggest that a 3 flip angle design may be most efficient both in terms of T_1 mapping efficiency and minimum number of required measurements. Several practical 3 flip angle designs for several repetition times are given in Table 2. Figure 2 shows comparison of our method with method of Koay et al. [6].

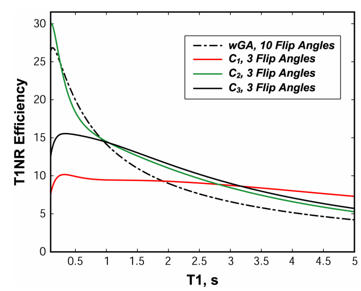

Figure 1. T_1NR efficiency curves from maximizing several optimization criteria. Mean efficiency gains from proposed designs (3 flip angles) relative to wGA curve (10 flip angles) are given by 1.14 (C1), 1.19 (C2), 1.22 (C3) (T_R =5 ms)

Discussion: We have presented a method for automated selection of flip angles for optimum performance of SPGR-based T_1 mapping over a wide range of T_1 values. We found that excellent efficiency could be gained with only 3 flip angles. The method outperformed both the 10 flip angle wGA and the 3 flip angle designs of [6] (Figs. 1,2).

Our results are different from those of Koay et al., where flip angles were clustered around only 2 values; our studies showed clustering around 3 values. It is clear from Fig. 2 that narrowing the optimization range improves the performance of Koay's method. Hence, we suggest that Koay's' method may be useful for narrower T_1 ranges. Another difference is that the optimization of Koay's targeted the variance of T_1 estimates, i.e., absolute error. Conversely, we targeted the optimization of relative errors in a T_1 range which focuses on the precision. It is interesting to note that the C_2 -design (2,8,19) was close to the empirical design in [5] obtained combining similar angles from the set of ideal flip angles for 2 representative T_1 values from the range. In general, such a heuristic search may not always be well posed, if ideal flip angles are not clustered. For example, the set of the ideal flip angles is not clustered for TR=20 ms (3,7,17,38). Using an automatic search as proposed here enables an optimized design with flip angles (3,14,38).

The criteria C_2 and C_3 introduced here demonstrated excellent performance for optimum designs. Surprisingly, they are not used as widely as C_1 [7,8]. Particular choice of a design criterion should be guided by application requirements. C_2 and C_3 are best at maximizing efficiency, while C_1 and C_3 provide more uniform response. This optimization framework may be useful for optimization of any quantitative MRI technique.

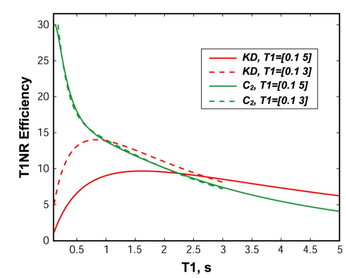

Figure 2. Comparison of T_1NR efficiency curves for new design (2,8,19) and method of Koay [4] (*KD*) (2,10,10) for two T_1 ranges.

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	C_1	C_2	C_3
3 FA	FA=(1,7,10) *v*=7.29	FA=(2,8,19) *v*=5.34	FA=(2,8,12) *v*=1.92
10 FA	FA=(1,6,11) *w*=(5,4,1), *v*=7.74	FA=(2,8,18) *w*=(4,3,3), *v*=5.41	FA=(1,2,8,9,16) *w*=(3,2,4,1,1), *v*=2.0

Table 1. Comparison of several designs for T_R = 5 ms (FA – flip angles, *w*-number of repetitions, *v*-final value of optimization flip angles).

TR	C_1	C_2	C_3
10 ms	(2,8,13)	(2,10,28)	(2,10,19)
15 ms	(2,10,16)	(3,13,33)	(3,13,22)
20 ms	(2,12,19)	(3,14,38)	(3,15,26)

Table 2. Practical optimized T_1 mapping experiment designs.