Understanding Acoustic Noise Suppression with Gradient Design: A Vibrating String Model

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Introduction Vibrations of the gradient coil, and of the surrounding conducting material where eddy currents are produced, lead to acoustic noise that is a major physical consequence of MRI operations and largely undesirable. More rapid switching of the gradients leads to even more pronounced vibrations. The origin of the noise thus brings up the possibility of using the same source to cancel the vibrations as that which has produced them: by modifications of the gradient pulse sequence. Within a spring model, vibrations resulting from an impulsive force can be cancelled immediately upon the application of another impulsive force [1]. This cancellation can still be effected even in the presence of damping; practically a continuum of follow-up impulse forces and timings can be implemented. In order to consider more closely the standing wave spatial aspects of the vibrations it is useful to investigate a forced and damped vibrating string, which previously has been put forth as a model of the dominant frequency for both longitudinal and transverse gradient coils [2]. Since the gradient vibrational modes yield a rich spectrum of frequencies (a significant frequency response over a range of several kHz), the present paper is on the application of the string model for any given frequency (peak and decay width) in the spectrum, and to account for previous experiments on peak cancellation [3]. Of special interest is the use of the string analytical solutions for a given frequency as a guide to find out how to kill that vibrational mode after it has been excited. The general solution of the string model is a robust embodiment of a linear response system [4] such as the MRI gradient coil.

General String Solution and Cancellation Mechanisms While the analytics are rather detailed, it is possible to write a complete damped solution: If the wave velocity is υ, the driven and damped (κ) string equation, $\frac{\partial^2 y(x,t)}{\partial x^2} + \frac{2\kappa}{\upsilon^2}\frac{\partial y(x,t)}{\partial t} - \frac{1}{\upsilon^2}\frac{\partial^2 y(x,t)}{\partial t^2} = -f(x,t)$, has the solution

$$y(x,t) = \int_0^l dx_0 \int_{-\infty}^t f(x_0,\tau) G(x,x_0,t-\tau), \text{ with } G(x,x_0,t-\tau) = \frac{2\upsilon^2}{T}\sum_{n=1}^{\infty}\frac{\sin(\pi n x/l)\sin(\pi n x_0/l)}{2\pi\upsilon_n} e^{-\kappa(t-\tau)}\sin(2\pi\upsilon_n(t-\tau)) \text{ for } t > \tau$$

and n[th] harmonic frequencies $\upsilon_n = \frac{n\upsilon}{2l}\sqrt{1-\left(\frac{\kappa l}{n\pi\upsilon}\right)^2}$ [5]. To introduce the cancellation mechanism in a clear fashion, consider the solution for the κ=0 case, and $f = f_0$, a constant for $0 < t < t_1$ (zero otherwise): $y(\kappa=0, x, t > t_1) = f_0 \frac{2\upsilon^2}{T}\sum_{odd\ n}\sin(\frac{n\pi x}{l})\frac{2l}{\pi}\frac{1}{\omega_n^2}[\cos\omega_n(t-t_1) - \cos\omega_n t]$.

where it is observed that every term is zero for $\omega_1 t_1 =$ any multiple of 2π forcing the vibration set forth from the (instantaneous) ramp up of the boxcar to be cancelled by the "anti-vibration" produced by the boxcar ramp down. Alternatively, for shorter multiple pulses, the "ramp up vibration" can be cancelled by the ramp up of another pulse with appropriate sign and timing, and so on. This mechanism still holds when we include damping, and it can be shown that the ramp and flat-top times of a trapezoidal gradient can be adjusted to kill two frequencies and, for a pulse generated by a series of boxcar convolutions, every additional convolution can be adjusted to kill an additional frequency. An alternative is to use multiple pulses: pairs are needed to null an additional frequency, pairs of pairs can be used to kill two additional frequencies, etc. The vibration frequency to be nulled determines the time required between the ramp and the "anti-ramp" for all cases.

Results and Previous Experiments Numerical solutions of strings driven by pulse sequences and their Fourier analysis are consistent with the above results. Any given frequency excited by a gradient pulse ramp-up can be cancelled by a follow-up ramp-down (either within the same pulse or as part of a follow-up pulse). Figure 1 illustrates the cancellation along with a timing that can actually enhance the vibration and an intermediate timing. The dominant peak heights in the acoustic response spectrum have been described as "fluctuating" periodically with respect to the trapezoidal gradient impulse flat-top width, "strongly suggesting that mechanical resonances of the gradient coil structure are the source of this acoustic energy" [3] The present study confirms this connection and, moreover, shows the fluctuations to be sinusoidal. See Figure 2. Finally, consider an example of multiple peak suppression, which is desired for sound suppression. A solution with a pair of trapezoid pulses, and timings of ramp t_r, flat-top t_{top}, gap t_g, leads to a cancellation of three frequency peaks at $1/t_r$, $1/(t_r + t_{top})$, $1/(2t_r + t_{top} + t_g)$. All of the examples utilizing this cancellation mechanism, including the damping effects and the different spatial excitations for longitudinal and transverse gradients, have been verified by a numerical solution of the string equation. While an optimized solution for noise suppression within the constraint of imaging pulse sequences remains to be investigated, we have found a rich variety of gradient timings that may lead to significantly quieter MRI operation.

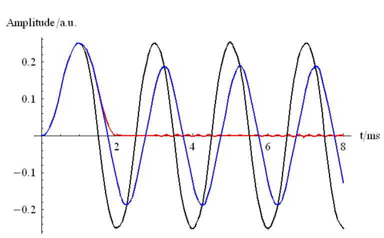

Figure 1

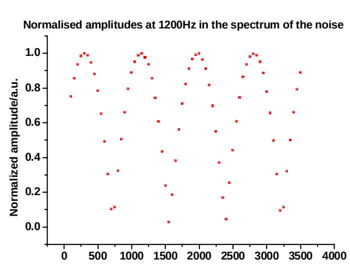

Figure 2

Figure 1: Simple boxcar timing illustration for a cancellation (red curve) with 2ms flat-top to kill a 500Hz peak. The black curve is a 1ms-top enhancement and the blue line is the 1.5ms-top intermediate result.

Figure 2: Vibration peak at 1200Hz driven with a trapezoid versus the flat-top time.

References [1] T. P. Eagan, et al., ISMRM 15, 1101, 2007. [2] D. Tomasi, T. Ernst, Brazilian J. of Phys., v. 36, no. 1A, 2006. [3] Y. Wu et al., MRM 44: 532-536 (2000). [4] A. Barnett, MRM 46: 207 (2001). [5] P. Morse and H. Feshbach, McGraw-Hill, 1953.