Transmission Line Effects on the Noise Correlation Matrix for Multiple RF Coils

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Introduction With the growing number of channels available on MR scanners, noise correlation is an important topic in RF coil design (1,2). The original noise correlation formula derived by Redpath (3) showed that correlation depends on the real part of the coil impedance matrix. In this work, we introduce a more generalized formula to include effects from transmission lines and preamplifier input impedance. The formula is used to calculate noise correlation between two-channel circuits connected to preamplifiers using several transmission line lengths.

Theory For a multiple coil system, the noise covariance is $\langle I\,I^*\rangle = 4kT\,\mathrm{Re}(\mathbf{Z}^{-1})$ or $\langle V\,V^*\rangle = 4kT\,\mathrm{Re}(\mathbf{Z})$ where $\mathbf{Z}$ is the coil impedance matrix (3,4). In an MR experiment, transmission lines are used to transfer signals from the coils to the preamplifiers. Correspondingly, the noise measured by the scanner can be determined from the noise detected by the coil according to transmission line theory. To demonstrate, consider a system of multiple coils connected to preamplifiers with finite input impedance (Fig. 1 shows a simplified two-coil system). Here the preamplifier impedance reflected to the coil is given by $Z_{a_{in}} = Z_0\left(1+\Gamma_a e^{-j2\beta l_a}\right)/\left(1-\Gamma_a e^{-j2\beta l_a}\right)$ where Z_0 is the characteristic impedance of the cable, l its electrical length, and $\beta = 2\pi/\lambda$ (5). The network equations are $V_a(-l) = \sum_b I_b(-l)\tilde{Z}_{ab}$ and $I_a(-l) = \sum_b V_b(-l)\left(\tilde{\mathbf{Z}}^{-1}\right)_{ab}$, where $\tilde{\mathbf{Z}}$ is a symmetric matrix which accounts for cable and preamplifier effects. Diagonal elements of $\tilde{\mathbf{Z}}$ are given by the sum of the corresponding coil self impedance and reflected preamplifier impedance $\left(\tilde{Z}_{aa} = Z_{aa} + Z_{a_{in}}\right)$, while off-diagonal elements are equal to the corresponding coupled impedance $\left(\tilde{Z}_{ab} = Z_{ab}\right)$. From Eq. [2.36b] in (5), current at the preamplifier is $I_a(0) = I_a(-l)(1-\Gamma_a)/\left(e^{j\beta l_a} - \Gamma_a e^{-j\beta l_a}\right) \equiv I_a(-l)\,f_a$. Therefore, the current noise correlation matrix at the preamplifiers is $\mathbf{\Psi} = \left\langle I_a(-l)\,I_b(-l)^*\right\rangle f_a f_b^* = f_a f_b^* \sum_{p,q}\left(\tilde{\mathbf{Z}}^{-1}\right)^H_{a,p}\left\langle V_a(-l)V_b(-l)^*\right\rangle\left(\tilde{\mathbf{Z}}^{-1}\right)^H_{b,q} = 4kT\; f_a f_b^*\,\tilde{\mathbf{Z}}^{-1}\,\mathrm{Re}(\mathbf{Z})\,\tilde{\mathbf{Z}}^{-1^H}$ [1], where H indicates the complex transpose.

Methods Noise measurements were carried out using three T–circuits with components chosen to provide predominantly real, imaginary, or complex mutual impedance (Table 1 lists $\mathbf{Z}$ recorded at 63.87MHz). The circuits were connected to standard preamplifiers on a 1.5T scanner (Excite; GE Medical Advances) using several transmission lines (l = λ/4, 3λ/8, λ/2, or 5λ/8). A 256×256 noise data set N was acquired from each channel by scanning with the RF excitation disabled. From this complex noise, the noise correlation matrix was calculated according to $\hat{\mathbf{\Psi}} = M^{-1}\sum_k N_{a_k} N^*_{b_k}$ where k is the acquisition matrix index of M points.

Results Good agreement between scanner noise measurements $\hat{\mathbf{\Psi}}$ and those predicted from impedance measurements $\mathbf{\Psi}$ (Eq. [1]) is demonstrated in Fig. 2. Fig. 3 shows the same data as that in Fig. 2, but plotted as a function of cable length to illustrate the distinct noise characteristics of each circuit. Note that the experimental measurements in Fig. 3 were normalized by the slope of the linear fit in Fig. 2.

Discussion We have introduced a general noise equation based on network circuit and transmission line theory. Eq. [1] shows that noise correlation at the preamplifiers depends on the real part of the coil impedance matrix $\mathbf{Z}$ as well as the transmission matrix $\tilde{\mathbf{Z}}$. This matrix can be modified through changes in cable length to achieve zero noise cross correlation Ψ_{ab} despite non-zero mutual impedance Z_{ab}.

Table 1. Coil impedance matrix measurements.

Circuit	Z_{aa} (Ω)	Z_{bb} (Ω)	Z_{ab} (Ω)
1	49.4+j3.1	49.2+j2.4	27.5-j3.2
2	48.4-j1.0	48.6+j0.9	2.7+j12.9
3	49.3+j2.2	49.4+j2.1	13.4+j15.6

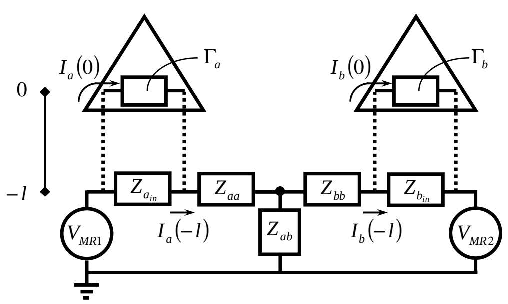

Fig. 1. Electrical schematic for a system of two coupled coils connected to preamplifiers of finite input impedance via transmission lines of arbitrary length.

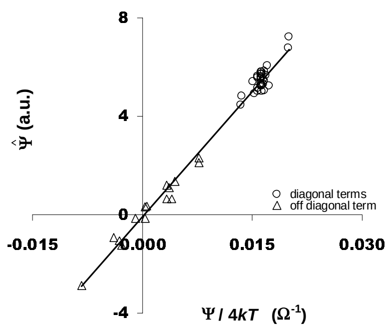

Fig. 2. Scatter plot of experimentally measured noise correlation (y-axis) versus the corresponding impedance evaluation by Eq. [1] (x-axis).

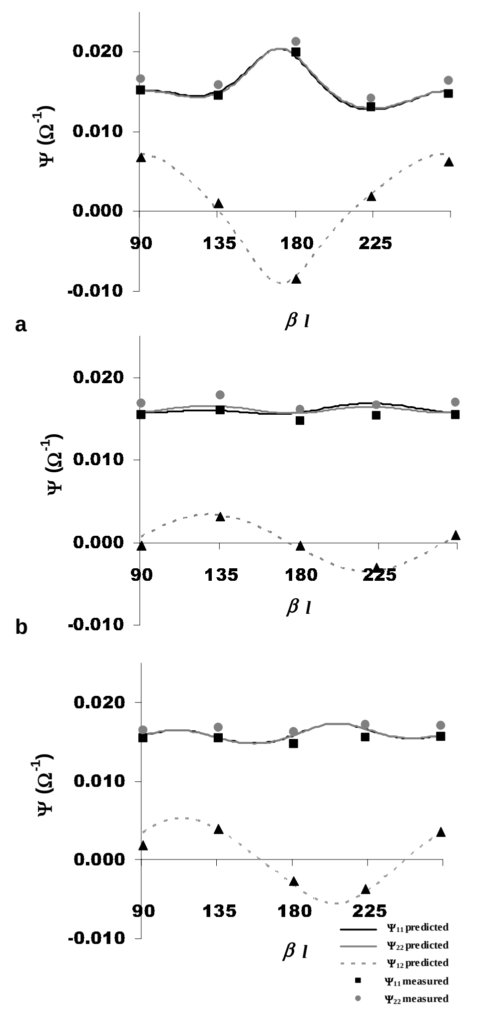

Fig. 3. Experimentally measured noise correlation versus transmission line length for circuits 1 (a), 2 (b), and 3 (c).

References 1) Ohliger MA, et al. MRM 2004;52:628. 2) Duensing GR, et al JMR B 1996;111:230. 3) Redpath TW. MRM 1992;24:85. 4) Brown R, et al. MRM 2007;58:218. 5) Pozar DM. Microwave Engineering. New York: Wiley; 2005.