Noise Measurement in Real-Time Cardiac Cine MRI Using Eigenvalues of Karhunen-Loeve Transform

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Introduction: Noise measurement is difficult in real-time cine MRI using parallel acquisition techniques because of cardiac and respiratory motion and the spatially variant nature of both signal and noise. Reeder et al proposed that the subtraction of two images can be used to measure the noise as long as the relative motion between them is not severe [1] (temporal subtraction method). However, the existence of two images containing exactly the same information is highly unlikely in dynamic cardiac image series acquired during free-breathing. In this work, we investigate using eigenvalues derived from the Karhunen-Loeve Transform (KLT) to assess mean noise standard deviation (SD) in dynamic MR cardiac images. This method may have general application in the evaluation of dynamic imaging methods, and other situations where temporal subtraction and other methods of noise measurement are difficult to apply.

Theory: KLT is a linear transform that exploits signal correlations to reduce high dimensional data into lower dimensions by transforming it into a set of orthogonal eigenmodes. The eigenvalue is the mean variance of the corresponding eigenmode. Dynamic MR images are highly correlated, thus most of the variance will be contained in the first few eigenimages corresponding to largest eigenvalues. The eigenimages associated with small eigenvalues contain mostly noise. Furthermore, the eigenvalue distribution asymptotically approaches the noise floor when the number of images approaches infinity [2]. According to random matrix theory [3], when the images contain only random noise, the mean noise variance would be equal to the median eigenvalue of the KLT transform. We propose that (1) the mean noise SD in dynamic images can be accurately estimated from the median eigenvalue when the number of images is much larger than the number of significant eigenimages; (2) when the number of images is comparable to the number of significant eigenimages, the minimum eigenvalue will provide a better estimation of the noise SD [4]. We compared the results with the temporal subtraction method. We used real-time cardiac cine images as an example in this study.

Methods: The study was performed on a 1.5T MRI system (Avanto, Siemens Medical Solutions, Inc). Six normal volunteers (age range from 25 to 54 with mean age 34) were imaged after giving written informed consent. Eight 250-frame real-time SSFP cines in the same mid-ventricular short axis view were acquired in each volunteer. Three cines with TSENSE acceleration factor 3, 4 and 5 were acquired using a standard 12-channel array. Other imaging parameters were: 192 x 144 matrix, 8.0mm thick slice, flip angle 68 degrees, TE = 1.02 ms, TR = 2.36 ms, pixel bandwidth=1370 Hz/pixel. In each image series, the 32 pairs of images with the highest cross-correlation coefficient were selected. The results of subtracting each of these highly correlated image pairs were evaluated visually to select the one showing the least amount of residual signal. The temporal subtraction method calculates the mean SD in this subtracted image divided by $\sqrt{2}$ as the mean noise SD of the corresponding image series [1]. The large number of images in the series and the method of choosing the best correlated pair ensured that the temporal subtraction method would provide an accurate measure of noise SD for comparison. When using the KLT for noise measurement, the mean noise SD equals to the square root of the median or the minimum eigenvalues. The relation of noise SD between the temporal subtraction method and the median eigenvalue method was analyzed using scatter plots and linear regression. In order to understand the influence of the number of images included on the two eigenvalue methods, we also varied the number of images used to assess noise SD by the eigenvalue methods, and compared them to the temporal subtraction method. The evaluation was repeated on images acquired with different acceleration factor. The data from 6 volunteers were averaged to form one curve.

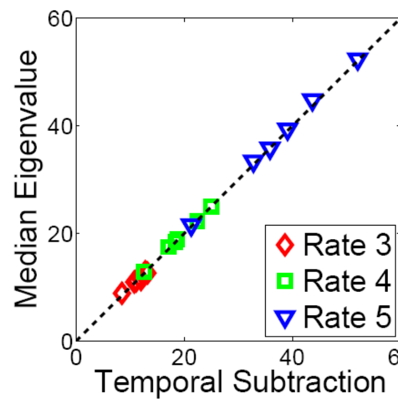

Figure 1. Scatter plot of the noise standard deviation obtained by temporal subtraction vs. median eigenvalue. The dashed line is the line of identity. By linear regression: y = 1.0086*x with $R^2 = 0.9993$.

Results: Figure 1 compares the noise SD assessed by temporal subtraction and median eigenvalue. The linear regression between the two measurement methods indicates that the difference is less than 1.0%. Figure 2 shows how the noise SD from the eigenvalue methods changes with the number of images, relative to the noise SD found from the temporal subtraction method. The median eigenvalue method tends to perform better in low SNR images (high TSENSE acceleration rate), and when more images are included (less than 10% deviation when more than 60 images are included, spanning approximately 4 heat beats). When less images are used, the minimum eigenvalue method tends to perform better (less than 10% deviation when less than 60 images are included).

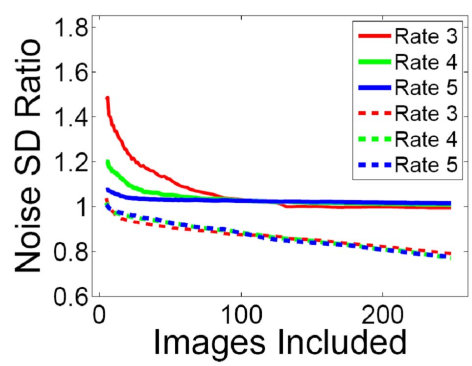

Figure 2. Averaged noise SD ratio from the eigenvalue methods vs. the number of images, relative to the noise SD found from the temporal subtraction method. Solid lines: noise SD ratio of median eigenvalue and temporal subtraction methods; dashed lines: noise SD ratio of minimum eigenvalue and temporal subtraction methods.

Conclusion: We demonstrate the effectiveness and accuracy of an alternative noise measurement method based on the eigenvalues of the KLT. Linear regression between the noise SD generated by this new method and the temporal subtraction method showed a good agreement. Unlike temporal subtraction, the KLT method is automatic. It relies on neither the existence of two exactly identical images, nor on subjective judgment. Furthermore, it can be applied to any dynamic image series, such as dynamic contrast MRI or echocardiography. However, the new method does show non-trivial deviations when fewer images are included in the original series because the true eigenvalue distribution of an image set with an arbitrary number of images is still unknown. Our future study will focus on applying the random matrix theory to understand the relation between the KLT eigenvalue distribution and noise properties, and develop a better algorithm to assess noise SD based on the eigenvalue distribution.

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