

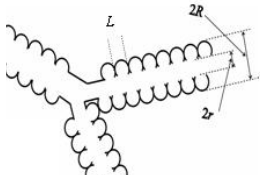
# A theory of diffusion time dependence of ADC in hyperpolarized $^3\text{He}$ lung MRI. Millisecond range

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**Introduction:** Lung diffusion MRI with hyperpolarized  $^3\text{He}$  gas provides information on lung structure and function. Substantial increases in ADC have been reported in emphysema suggesting that  $^3\text{He}$  gas ADC could serve as a biomarker for disease progression. However, even in healthy human lungs ADC exhibits rather broad variability, with different studies reporting results between  $0.15 \text{ cm}^2/\text{s}$  and  $0.25 \text{ cm}^2/\text{s}$ . Here we provide a theoretical analysis demonstrating a significant diffusion time dependence of ADC, even for diffusion times on the order of several milliseconds, which can explain the observed ADC variability.

**Theory:** The *in vivo* lung morphometry technique (1) allows evaluation of lung microstructure based on MRI measurements of the diffusion of hyperpolarized  $^3\text{He}$  gas. According to the lung geometric model (2) adopted in (1), an acinar airway is considered as a cylinder covered by a sleeve of alveoli (see Fig. 1). In humans, depending on the branching level of the acinar airway tree, the internal acinar airway radius  $r$  varies in the interval from  $135 \mu\text{m}$  to  $250 \mu\text{m}$ , whereas the outer radius  $R$  (including the sleeve of alveoli) remains practically constant at  $350 \mu\text{m}$  (2).  $^3\text{He}$  atoms can freely diffuse within the internal cylinder of radius  $r$ ; however, the alveolar walls (internal and external) are treated as impermeable to the gas atoms. Hence, diffusion of  $^3\text{He}$  gas in each airway is anisotropic and described by distinct longitudinal and transverse diffusion coefficients,  $D_L$  and  $D_T$ . An analytical expression for the diffusion MR signal as a function of  $b$ -value was derived taking into account that a multitude of airways are present in each imaging voxel, with a uniform distribution of orientations. This model allows estimation of  $D_L$  and  $D_T$  from multi  $b$ -value MR experiments (1). In most experimental studies of gas diffusion in the lungs, only the mean isotropic ADC is determined from measurements with small  $b$ -values. In this limit, ADC has a simple relationship to  $D_L$  and  $D_T$  (1):



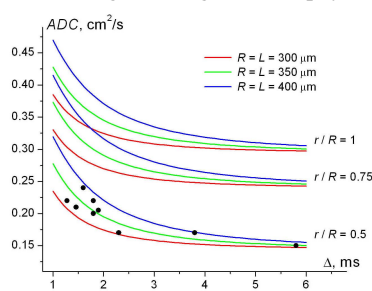
$$ADC = (D_L + 2D_T) / 3 \quad [1]$$

In the millisecond range of diffusion times, the parameters  $D_L$  and  $D_T$  are related to the geometrical parameters of acinar airways  $R$ ,  $r$ ,  $L$  by the following empirical expressions (3):

$$D_L = D_0 \cdot \left\{ 1 - (R/L)^{1/2} \cdot \left[ 1 - \exp\left(-2.5 \cdot (1 - r/R)^{1.8}\right) \right] \right\}, \quad D_T = 0.44 D_0 \cdot (R/L_{diff})^4 \cdot \left[ 1 - (R/L_{diff})^{0.7} \right] \quad [2]$$

Here  $D_0$  is the quasi-free diffusion coefficient of  $^3\text{He}$  gas in lung airspaces;  $L_{diff} = (4D_0\Delta)^{1/2}$  is the characteristic free-diffusion length for two-dimensional diffusion. Equations [2] are valid for  $R/L_{diff} < 0.5$  and  $r/R > 0.4$ . Eqs. [2] predict that the longitudinal diffusivity  $D_L$  does not depend on the diffusion time  $\Delta$ , whereas the transverse diffusivity  $D_T$  decreases as  $\Delta$  increases. In the limiting case  $R \ll L_{diff}$ ,  $D_T \rightarrow 0$  and ADC is completely determined by the longitudinal diffusivity  $D_L$ :  $ADC \approx D_L / 3$  and is independent of  $\Delta$ . This result is valid only when  $(2D_L\Delta)^{1/2}$  is smaller than the mean length of alveolar ducts ( $\sim 1 \text{ mm}$ ); hence, assuming a typical  $D_L \sim 0.4 \text{ cm}^2/\text{s}$ , the diffusion time should be less than 12 ms. For longer diffusion times, a theory accounting for branching of acinar airways is required (4). Although  $D_T$  is smaller than  $D_L$  for  $R \sim 300\text{-}400 \mu\text{m}$  (radii characteristic to human lungs) and  $\Delta$  about several milliseconds, its contribution to variations in ADC is substantial.

**Results and Discussion:** Figure 2 illustrates the dependence of ADC on  $\Delta$  for different external radii  $R$  and ratios  $r/R$  (colored lines) obtained from Eqs. [1,2] with  $D_0 = 0.88 \text{ cm}^2/\text{s}$ . Note the significant dependence of ADC on  $r/R$ , whereas its dependence on  $R$  is comparably weak. Black dots in Fig. 2 correspond to ADCs experimentally found in healthy human lungs, found for different  $\Delta$  in (5). Also we included single- $\Delta$  results from (1, 6-10). All of these ADCs fall within an interval of  $R$  around  $350 \mu\text{m}$  (in good agreement with histological data (2)) and correspond to  $r/R \approx 0.5$ . The dependence on diffusion time  $\Delta$  of this data is nicely explained by Eqs.[1]-[2]. The experimental results presented in Fig. 2 correspond to healthy human lungs. In lungs with emphysema, the ADC is higher. The increase of ADC at initial stages of emphysema (ADCs  $\sim 0.3\text{-}0.4 \text{ cm}^2/\text{s}$  for  $\Delta \sim 1\text{-}2$  ms) could be attributed to increasing the ratio  $r/R$  (internal alveolar walls are being destroyed). In lungs with severe emphysema, ADCs  $\sim 0.6\text{-}0.7 \text{ cm}^2/\text{s}$  were observed, e.g., (1, 4), which is significantly higher than predicted by Eqs. [2], even for  $r/R=1$ . At this stage of disease, the external walls of airways are being destroyed and the ‘‘cylindrical’’ model of airways becomes invalid.



In small rodents, the radius  $R \sim 100\text{-}200 \mu\text{m}$ , transverse diffusion is very restricted and  $D_T$  is very small. In this situation,  $ADC \approx D_L / 3$ . This result is in a good agreement with experimental data obtained in rat lungs (11):  $D_L = 0.3 \text{ cm}^2/\text{s}$ ,  $D_T = 0.01 \text{ cm}^2/\text{s}$ ,  $ADC = 0.11 \text{ cm}^2/\text{s}$ . This value of  $D_L$  corresponds to  $r/R = 0.37$ . To observe time dependence of ADC in the lungs of small animals, diffusion times less than 0.5 ms are needed.

**Conclusion:** Experimentally detected variability of  $^3\text{He}$  ADC in healthy human lungs can be explained by the substantial dependence of ADC on diffusion time, even over the narrow millisecond range.

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