An improved algorithm for data reduction prior to independent component analysis of functional MRI data in the presence of colored noise and low source signal-to-noise ratio

V. J. Schmithorst[1], and S. K. Holland[1]

[1]Pediatric Neuroimaging Research Consortium, Radiology, Children's Hospital Medical Center, Cincinnati, OH, United States

Introduction

Data reduction (usually performed via Principal Component Analysis, PCA) has been shown to be a critical step for accurate blind-source-separation in noisy data [1], performed via a method such as Independent Component Analysis (ICA). Without data reduction (separating the data space into a signal-plus-noise subspace and a noise-only subspace), the ICA algorithm may get "stuck" in a local optimum and fail to successfully unmix the sources [1]. The correct number of sources must be retained in the signal-plus-noise subspace, thus producing a model order estimation problem. Too many sources retained may result in unsuccessful unmixing, while too few sources retained will result in failure to detect the sources excluded from the subspace. For ICA analysis of functional MRI (fMRI) data, the problem is exacerbated by the fact that noise in fMRI data is not "white", but "colored" (e.g. having greater contribution at lower frequencies). This makes PCA a sub-optimal method for data reduction, and also renders techniques such as Bayesian model evidence optimization [2], which rely on the assumption of i.i.d. noise, less robust. A data reduction method has been published [3] which uses an AR(1) model of the noise. The method has been demonstrated to more accurately estimate the model order as compared to conventional techniques, and has the desired quality of the estimate model order not increasing with number of fMRI frames acquired. However, the method severely underestimates the model order at low SNR of the sources. Here we present a technique for accurate estimation of the model order and extraction of the source subspace in the presence of colored noise and low source SNR.

Theory

Assume the fMRI data has been grouped into a 2-dimensional *T*-X-*N* matrix, with *T* the number of time frames and *N* the number of voxels. In the method of Cordes and Nandy [3], standard PCA is performed and the tail of the resulting eigenspectrum fit to the following function:

$$\lambda(k) = a\exp(-bk) + \Delta \qquad (1)$$

where $\lambda(k)$ is the k^{th} eigenvalue, and Δ is a parameter reflecting a shift in the eigenspectrum resulting from variance normalization of the time courses. Reference values for a and b (for a given T and N), for varying values of the AR(1) coefficient ϕ, are obtained via simulation. The actual value of ϕ is estimated by comparison to the reference values. The real eigenspectrum is compared to the theoretical noise eigenspectrum (computed using the above equation), and the model order estimated by determining where the values of the real eigenspectrum are greater than the noise eigenspectrum. The method has been shown to outperform conventional model order estimation techniques, such as Minimum Description Length (MDL) or model order evidence [2]. However, when the sources have low SNR the real eigenspectrum fits the noise eigenspectrum quite well even for small eigenvalue numbers (Figure 1, left). This effect is due to the fact that for non-zero ϕ standard PCA is a sub-optimal method of data reduction.

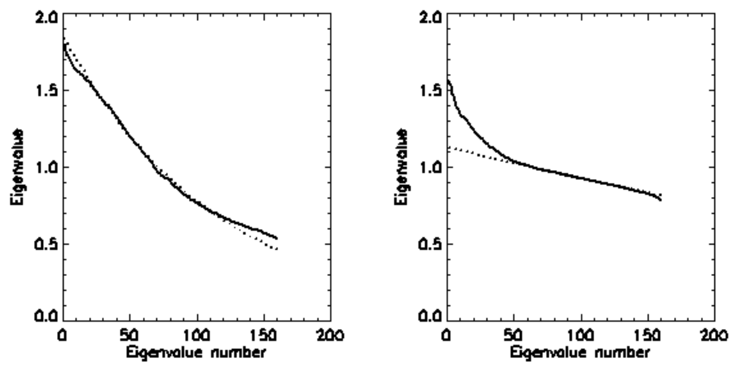

Figure 1. Comparison of theoretical (dashed, taken from Ref. [3]) and empirical (solid) eigenspectrums for ϕ = 0.25, T = 160 time frames, p = 50 sources, N = 20,000 voxels, SNR = 0.5, computed before (left frame) and after (right frame) prewhitening.

Therefore we propose a slightly different technique suitable for correct model order estimation even at low SNR. Empirically, b is a minimum for lower values of ϕ (e.g. less autocorrelation in the noise). Thus, the data is "pre-whitened" via pre-multiplying with the inverse of the Cholesky decomposition of the noise covariance for a given ϕ and standard PCA performed on the pre-whitened data. The eigenspectrum is fit to equation (1) and the optimal value of ϕ determined by minimization of b. After the data is pre-whitened, there is significantly better separation between the noise and the real eigenspectrums (Figure 1, right). Since the estimate of ϕ may not be exact, and the noise model may vary from AR(1), to determine the model order the coefficient of determination of fitting equation (1) to the putative noise eigenspectrum is computed, and compared to a simulated distribution of coefficients of determination for given *N* and *T* in order to estimate the log-likelihood function. An Aikake's Information Criterion (AIC)-like cost function is computed to optimize the model order.

SNR	# Sources Method 1	# Sources Method 2	R	R_{MAX}
0.3	2	20	0.21	0.31
0.4	3	41	0.34	0.39
0.5	4	45	0.43	0.46
0.75	12	51	0.59	0.60
1.0	22	51	0.69	0.70

Table 1. Comparison of performance of the original data reduction methodology (Method 1, from Ref. [3]) and proposed methodology (Method 2) for ϕ = 0.25, T = 160 time frames, p = 50 sources, N = 20,000 voxels. (R = correlation of found sources from Method 2 with ground truth, R_{MAX} = maximum possible correlation).

Materials and Methods

Datasets were simulated using routines written in IDL (Research Systems Inc., Boulder, CO). Background noise was simulated from a Gaussian distribution, with an AR(1) coefficient ϕ of 0 to 0.25. 20000 voxels were used and the number of time points was 160. 50 sources were simulated using a Laplace distribution and mixed into the noise using a random mixing matrix, with SNRs varying from 0.3 to 1.0. The model order was estimated using the modified technique described above. The extracted sources were then projected onto the subspace of (ground-truth) sources and the correlation coefficient computed as a measure of accuracy of source extraction, compared to the correlation coefficient obtained by projecting the entire dataset onto the source subspace (the maximum obtainable).

Results

At low SNR and high autocorrelations, the proposed method performs very well (Table 1) relative to the previously published method [3], which for the same *T* and *N* and similar ϕ significantly underestimated the model order at low SNR. The autocorrelations are found exactly, and the model order is fairly accurately estimated for SNR of 0.4 and above.

Conclusion

A new method for data reduction prior to ICA of fMRI data is proposed. The method is particularly suitable for datasets where there is high intrinsic autocorrelation in the noise and low SNR in the sources. Future research (data not shown) will investigate inclusion of a higher-order term in the exponential of equation (1) to better model eigenspectrums from more highly colored noise (e.g. $\phi > 0.4$)

References

[1] Hyvarinen A. Independent Component Analysis. John Wiley & Sons, 2002. [2] Minka T. P. *NIPS*, 13, 598, 2000. [3] Cordes D, Nandy R. *Neuroimage*, 29, 145, 2006.