Sparse Decoding in fMRI

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Introduction

"Sparsity of information" in data has been observed in many areas and exploited in various applications such as compression (JPEG), denoising [1] and fast data acquisition (compressed sensing in MRI [2,3]). In fMRI, there exist a large number of sources that affect the time course of a voxel. These include evoked and spontaneous brain activity, respiration, heart beat, head motion, scanner instability and other regional or global sources. However, a typical voxel time course is affected by only a subset of these sources (Fig. 1) and provided that the number of sources is small (e.g. ~ 15), one might be able to effectively model them to improve their identification and extract relevant information from the data. Here we explored this idea on fMRI data.

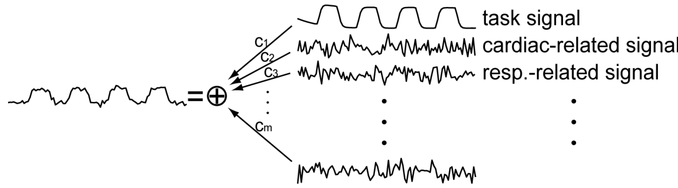

Fig. 1: A voxel time course can be considered as the combination of a few sources out of a large number of sources (i.e. most of $c_k = 0$)

Sparse Decoding

Based on our model, the fMRI data matrix ($\mathbf{V} = [\mathbf{v}_1\ \mathbf{v}_2 \ldots \mathbf{v}_n]$, where $\mathbf{v}_i$ is the time course of i^{th} voxel) can be decomposed into a source matrix ($\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2 \ldots \mathbf{s}_m]$, t time points with m sources where $m > t$), and a sparse weight matrix ($\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2 \ldots \mathbf{w}_n]$, a m x n matrix) (Fig 2). Our task is to find the source matrix that makes the sparse weight matrix as sparse as possible. This can be formulated as follows:

$$\min_{\mathbf{S},\mathbf{W}} \sum_{i=1}^{n} \|\mathbf{v}_i - \mathbf{S}\mathbf{w}_i\|_2^2 \quad s.t.\ \ \forall i, \|\mathbf{w}_i\|_0 < L$$

where L is the maximum number of components in a sparse weight vector (from our assumption $m >> L$), and $\|.\|_0$ represents L_0 norm which counts the number of components in a vector. A two-step approach can be deployed to solve this problem. First, for a given source matrix, the sparse weight matrix is found column by column using the sparse coding:

$$\mathbf{w}_i = \arg\min_{\mathbf{w}_i} \|\mathbf{v}_i - \mathbf{S}\mathbf{w}_i\|_2 \ \ s.t.\ \ \|\mathbf{w}_i\|_0 < L$$

This can be solved using a pursuit algorithm [4]. Once the sparse weight matrix is found, the source matrix is updated column by column. When the k^{th} source vector ($\mathbf{s}_k$) to be updated, all the other vectors in **S** are frozen along with all the **w** rows except for the k^{th} row ($\mathbf{w}^T{}_k$). Then a new $\mathbf{s}_k$ and $\mathbf{w}^T{}_k$ that minimize $\|\mathbf{V} - \mathbf{S}\mathbf{W}\|_2$ can be found using a singular value decomposition (SVD). The whole process is iterated to reduce the total error. This method is called K-SVD [5] since it is a generalization of the K-mean [6] combined with the SVD. The resulting source matrix will represent the fMRI data with the sparsest combination.

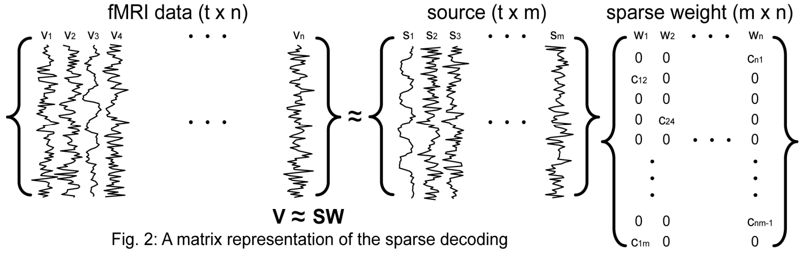

Fig. 2: A matrix representation of the sparse decoding

Experiment and Results

To validate this sparsity model for fMRI data, the sparse decoding was performed for two different types of datasets: 1) block design fMRI data (6 subjects, 4 with a visual stimulus paradigm, 1 with visual and motor, and 1 with visual, auditory and motor stimuli, 3.75 x 3.75 x 5 mm^3, # slices = 20 to 30, TR = 2 to 3 s, total scan time = 3 to 5 min, 3 T), and 2) resting-state data (3 subjects, eyes open, 3.75 x 3.75 x 5 mm^3, # slices = 30 slices, TR = 3 s, total scan time = 5 min, 3 T). The results were compared with a GLM (FEAT, FSL) for the task-evoked activity data and an ICA (MELODIC, FSL) for the spontaneous activity (resting-state) data. For the sparse decoding, the parameters were set to m = 150, L = 15, and # iteration = 100. To find the weight matrix, the matching pursuit algorithm [4] was used. For the comparison with conventional task activation fMRI analysis, the source vectors were correlated with the task regressor from the GLM. The activation map of the GLM and the sparse map (the spatial map of a given source vector) of the highest task-correlated source were compared. The results show that the mean correlation (6 subjects) between the maximally task-correlated source vectors and the task regressor was 0.86 ± 0.04. The activation map and the sparse map show a good correspondence (Fig. 3). In the resting-state data, the sparse decoding results (Fig. 4) show similar spatial patterns as found with ICA (results not shown).

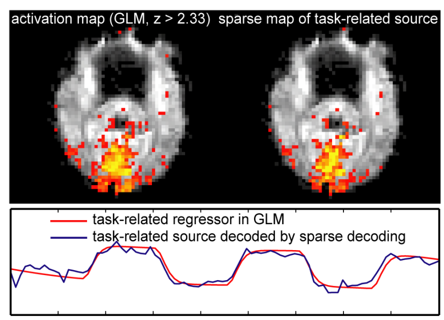

Fig. 3. The activation maps from GLM vs sparse decoding

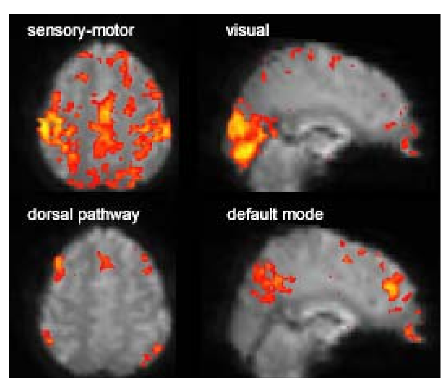

Figure 4. Correlation maps of four sources of restingstate networks from sparse decoding

Discussion and Conclusion

Using the sparse decoding, the task-evoked activation maps and the resting-state maps were successfully retrieved. Hence the sparsity model is potentially applicable to fMRI data. Compared to ICA, this method has a few different features. First, it is more flexible in choosing the decomposition parameter such as m and L as long as $n > m >> L$ and $m > t$. Another potential advantage is that one could include known regressors into the source matrix for a better decomposition (provided valid regressors are used). Also, when two or more source vectors are similar, one could merge them into one if necessary [5]. This merging process could be useful in improving the over-fitting problem.

References

[1] Donoho, Comp. Rend. Acad. Sci., 1994, 319:1317 [2] Lustig, ISMRM, 2006, p. 695 [3] Ye, MRM, 2007, 57:764 [4] Mallat, IEEE TSP, 1993, 41:3397 [5] Aharon, IEEE TSP, 2006, 54:4311 [6] Dempster, J. Roy. Statist. Soc., 1997, 39:1