Multi-Shelled Q-Ball Imaging: Moment-Based Orientation Distribution Function

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Background and Purpose

q -ball imaging

q -ball imaging (QBI) method calculates the orientation distribution function (ODF) that represents a kind of magnitude of diffusion in each direction (1). The ODF is useful for the estimation of the neurofiber orientation; in particular, there are cases that we can detect the direction of individual fiber group even if the multi fiber groups exist in a single voxel. The directions are given as ones of local maxima of the ODF profile, however, there are some mismatch between them in general (2). For the case that two fibers cross in a voxel with an acute intersection angle, the angle estimated by the ODF is, in general, smaller than the actual angle (Fig. 1), and the local maxima of the ODF profile indicate only one orientation when the intersection angle between the fibers becomes smaller than a threshold.

QBI is a high-angular-resolution diffusion image and demands a large number of data acquisition measurements. Reduction of the number of the measurements is a desired improvement for QBI together with the betterment of the angular resolution.

q -space imaging using small magnetic field gradient

q -space imaging (QSI) method gives the probability density function (PDF) of diffusion displacements. From the PDF we can extract several quantities that indicate characteristics of the diffusion. We have ever pointed out that we can obtain some of these quantities by precise measurements of the signal intensity in low q region without making PDF (3) as explained below. The n th order moment of PDF is defined as $E(R^n)=\int_{-\infty}^{\infty}P(R)R^n dR$, where R is the diffusion displacement and $P(R)$ is PDF. The Fourier relation between PDF and the diffusion signal $S(q)$ is $S(q)=\int_{-\infty}^{\infty}P(R)\,e^{iqR}dR$. Thus, we have $E(R^n)=\left(\frac{1}{i}\frac{d}{dq}\right)^n S(q)\,|_{q=0}$, i.e., we can obtain the n th order moment by knowing the n th order differential coefficient of $S(q)$ at $q=0$, which can be known from the behavior of $S(q)$ when we change the q near 0. The mean displacement and the kurtosis are obtained from $E(R^2)$ and $E(R^4)$.

Purpose

We propose a method that yields moment-based ODF. This method is based on QBI and the idea of "QSI using small magnetic field gradient". We also investigate the ability of the ODF to identify the fiber crossing and the possibility of reduction of the number of the measurements.

Multi-shelled QBI

In cylindrical coordinates, the 3-dimensional Fourier relation $S(\vec{q})=\int_{-\infty}^{\infty}P(\vec{R})\,e^{i\vec{q}\cdot\vec{R}}d^3R$ ($\vec{q}$: q vector, $\vec{R}$: diffusion displacement vector) becomes

$$S(q_\rho,q_\theta,q_z)=\int_0^{\infty}dR_\rho\int_0^{2\pi}dR_\theta\int_{-\infty}^{\infty}dR_z R_\rho P(R_\rho,R_\theta,R_z)\exp\left[i\{q_\rho R_\rho\cos(R_\theta-q_\theta)+q_z R_z\}\right], \quad [1]$$

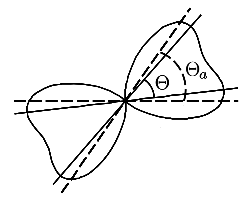

FIG. 1. An ODF profile

where the subscripts ρ and θ denote the vector components of the radius and the azimuthal directions, respectively. By setting $q_z=0$ and $q_\rho=q$ (a radius of q -ball), and integrating out with respect to q_θ, we have

$$\int_0^{2\pi}S(q,q_\theta,0)\,dq_\theta=\int_0^{\infty}dR_\rho\int_0^{2\pi}dR_\theta\int_{-\infty}^{\infty}dR_z R_\rho J_0(qR_\rho)P(R_\rho,R_\theta,R_z), \quad [2]$$

where $J_0(\cdot)$ denotes the 0th order Bessel function. The quantity of the right-hand side of Eq. [2] is a value of the ODF for z -direction defined in the ordinary QBI. This quantity is obtained as the left-hand side of the equation. This is just the idea of the ordinary QBI.

Now, we consider the n th derivative of Eq. [1] with respect to q_z, and then set q_z to 0 and integrate out with respect to q_θ :

$$\left.\left(\frac{1}{i}\frac{\partial}{\partial q_z}\right)^n\int_0^{2\pi}S(q,q_\theta,q_z)\,dq_\theta\right|_{q_z=0}=\int_0^{\infty}dR_\rho\int_0^{2\pi}dR_\theta\int_{-\infty}^{\infty}dR_z R_\rho J_0(qR_\rho)P(R_\rho,R_\theta,R_z)R_z^n. \quad [3]$$

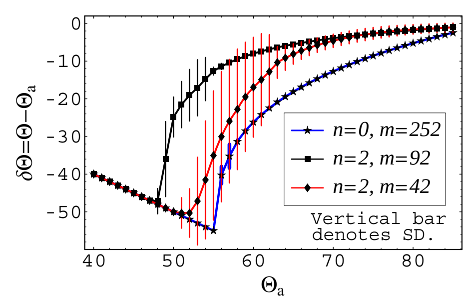

FIG. 2. Angular deviations

The quantity of the right-hand side of Eq. [3] is a kind of n th order moment of PDF with respect to R_z ; we define it as the value of ODF of our method. This quantity can be obtained as the left-hand side of the equation, which is, in practice, can be known from the behavior of $\int_0^{2\pi}S(q,q_\theta,q_z)dq_\theta$ when we change the q_z near 0. To obtain the values of the ODF of n th order moment for all directions, data acquisitions on $(n/2+1)$ shells of q -ball are necessary at least for even n . Figure 2 shows a result of a simulation of fiber crossing detections, where Θ and Θ_a are the angles between local maxima of the ODF and the intersection angle of actual two fibers, respectively. n denotes the order of the moment and $n=0$ corresponds to the ordinary QBI. m denotes the number of sampling direction, and the directions of $m=$ 252, 92 and 42 are obtained from the vertices of five-, three- and two-fold regularly tessellated icosahedrons projected onto the sphere, respectively. We have calculated the ODF of the second order moment by using two shells of the q -ball that radii are, in terms of b , 4000 s/mm ^{2} and 4800 s/mm ^{2} . The number of the measurements is $2m$ for $n=2$. The duration δ and the separation Δ of the gradient pulse we have assumed are 15 ms and 50 ms, respectively. The width of the spherical Gaussian interpolation kernel and the number of equator sampling points (1) are set according to Ref. 4. The diffusion characteristics of the fibers we have assumed are the same as Ref. 2. The results of Fig. 2 are the averages of 100 results obtained in randomly rotated fiber configurations that have respective fixed intersection angles, and the vertical bars denote the standard deviations for $\delta\Theta$'s (the mean errors are adequately small). The values of Θ_a when $\delta\Theta$'s begin to increase are the thresholds to detect the crossings. The results imply the possibility that the moment-based ODF can reduce the number of the measurements in keeping higher ability to identify the fiber crossing.

References

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