

# White Matter Tract Visualisation Using A Parabolic Eigensystem

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**Introduction:** Here we present a method for visual segmentation of tracts by interrogation of streamline termination coordinates and invocation of a parabolic eigensystem. This approach was designed to avoid problems associated with previous colour mapping schemes, in particular to avoid discontinuities in the colour mapping (e.g. 1) and reduction of computational time (e.g. 2). The method is applied to a normalized brain dataset comprising 30 healthy individuals. Visualisations of brain stem structures, thalamic structures and the recently described three segment model of the arcuate fasciculus are demonstrated. Additionally, this technique provides several geometric parameters that may be useful for streamline segmentation algorithms.

**Methods:** *MRI data acquisition:* 30 young healthy subjects were scanned on a 1.5T GE Signa MRI system (max. field gradient strength 22mTm<sup>-1</sup>). DTI was achieved using a single shot echo planar sequence with 12 diffusion sensitised directions as described previously (1). Two interleaved acquisitions comprising 25 slices each provided whole brain coverage (resolution: in plane 2.5mm; through plane 2.8mm). Each DTI was normalised to standard space by affine transformation. A mean DTI was computed.

*Tractography:* Subvoxel streamline DTT was performed on the mean DTI as described previously (1). Streamlines (vector step length 1.0mm, termination criteria FA < 0.1) were initiated from every voxel centre. Streamline termination coordinates  $t_1=(x_1 y_1 z_1)$  and  $t_2=(x_2 y_2 z_2)$  were computed relative to an image origin at the left, posterior, inferior corner and were normalized to have values between 0 and 1.

*White Matter Tract Visualisation:* This is achieved by creating a parabolic eigenvalue problem that has one non-zero real solution within a symmetric 3x3 matrix,  $A$  where,

$$A = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}, \text{ and the characteristic equation is}$$

$$\det(A - \lambda I) = -\lambda^3 + \lambda^2(a + d + f) + \lambda(b^2 + c^2 + e^2 - ad - af - df) + (adf - ae^2 - b^2f + 2bce - c^2d) = 0$$

The eigenvalues are parabolic with one non-zero eigenvalue if  $b^2 = ad$ ,  $c^2 = af$  and  $e^2 = df$ , and consequently  $\lambda^2(\lambda - (a + d + f)) = 0$ , giving non-zero real eigenvalue  $\lambda = a + d + f = \text{Tr}(A)$ . In terms of  $t_1$  and  $t_2$  parabolic conditions are  $a = x_1x_2$ ,  $d = y_1y_2$  and  $f = z_1z_2$  which give

$$A = \begin{pmatrix} x_1x_2 & \sqrt{x_1x_2y_1y_2} & \sqrt{x_1x_2z_1z_2} \\ \sqrt{x_1x_2y_1y_2} & y_1y_2 & \sqrt{y_1y_2z_1z_2} \\ \sqrt{x_1x_2z_1z_2} & \sqrt{y_1y_2z_1z_2} & z_1z_2 \end{pmatrix} \Rightarrow \lambda = x_1x_2 + y_1y_2 + z_1z_2$$

The associated eigenvector was mapped to an RGB colour (RED corresponds to  $x$  component, GREEN to  $y$  and BLUE to  $z$ ) and displayed at the tract seed voxel. This provides a visualisation of the white matter tracts in the human brain (Fig.1i). The eigenvectors associated with the repeated characteristic equation root,  $\lambda^2=0$  provide no further useful information (Fig.1ii). Interestingly the non-zero eigenvalue is equal to the scalar product between  $t_1$  and  $t_2$  (Fig.1iii).

**Results:** Figure 2 shows the utility of this technique by aiding visualization of different neuroanatomical pathways located in the pons (Fig.2i), cerebral peduncle (Fig.2ii), thalamus (Fig.2iii) and arcuate fasciculus (Fig.2iv) using different colours. Specifically, all labeled pathways in Figure 2i to 2iii were identified with comparison to (3) while pathways labeled in Figure 2iv were identified from (4). In particular, the visualisation technique provides a colour scheme for which the red component is larger when  $t_1$  and  $t_2$  are located further to the right of standard space. Similarly, the green and blue components are larger when  $t_1$  and  $t_2$  are located further to the anterior and superior of standard space. The utility of the technique is further demonstrated in Figure 3 where the colourmap is modulated by the eigenvalue (Fig.1iii) to visualise white matter pathways of the arcuate fasciculus from lateral (Fig.3i), posterior (Fig.3ii), superior (Fig.3iii) and medial (Fig.3iv) view points. Figure 3 clearly shows that the anterior, posterior and direct segments to be coloured differently and aids visualization of these pathways.

**Discussion:** We have presented a technique for visualising tract anatomy in standard space as inferred from streamline DTT. Although shown for mean DTI data this technique allows visual comparisons between similar white matter pathways delineated in multiple subjects. The stated technique overcomes problems associated with previous colour schemes (1,2). Firstly, it does not involve assignment of termination coordinates to  $t_1$  and  $t_2$  based on the location of the coordinates in standard space, a technical aspect of (1). Secondly, it does not have any of the problems associated with sharp discontinuities in the colour maps caused by interleaving the bits of two 4 bit numbers representing the termination coordinate  $x$ ,  $y$ , and  $z$  components to create 24bit RGB colours, a problem of (1). Finally it provides a similar colourmap to the Laplacian colour scheme (2) but at a much smaller computation cost due to the relative simplicity of the presented method. In addition, it is likely that these techniques will be useful in DTT image segmentation or registration as the eigenvalue and eigenvector, along with the angle between  $t_1$  and  $t_2$  (computed using the scalar product) provide unambiguous visual segmentation of white matter pathways.

## References

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