Interpolation and Regularization of Diffusion Tensors along Geodesics

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Introduction

The processing of diffusion tensor imaging (DTI) and tractography involve a number of steps, such as the registration, the estimation and the smoothing of tensor fields. These procedures implicitly rely on a measure to compare two or more tensors as well as on a continuous transformation of tensors. Here we propose a simple theoretical framework to separate the analysis of tensor fields into two parts: the first considers the shape of tensors, and the second their orientation. We illustrate the application of the interpolation and the regularization algorithms built from such a distance using a synthetic example and a DTI scan from a diffusion phantom.

Methods

For the definition of a distance between two positive definite tensors **A** and **B** we define $d(\mathbf{A},\mathbf{B}) = d_{shape}(\mathbf{A},\mathbf{B}) + d_{orient}(\mathbf{A},\mathbf{B})$, where the first measure compares the respective eigenvalues of the tensors, and the second quantity compares their eigenvectors. For the sake of simplicity we focus on axially-symmetric ellipsoids with $\lambda_2 = \tilde{\lambda}_3$. Shape is described by two quantities, the longitudinal diffusion $\lambda_{long} = \lambda_1$, and the transverse diffusion $\lambda_{trans} = \lambda_2 = \lambda_3$. They represent main biophysical properties of white matter fibers [1]. Orientation is represented by a single unitary vector **u**, and vectors **u** and **-u** represent the same orientation. The distance between two orientations can be defined as: $d_{orient}(\mathbf{A},\mathbf{B}) = \theta(\mathbf{A},\mathbf{B}) = \arcsin(\| \mathbf{u}_A \times \mathbf{u}_B \|)$, the acute angle ($0 \le \theta \le \pi/2$) formed by two orientations. This simple definition satisfies the following properties: $d_{orient}(\mathbf{u}, \mathbf{u}) = 0$, $d_{orient}(\mathbf{u}_1, \mathbf{u}_2) = d_{orient}(\mathbf{u}_2, \mathbf{u}_1) = d_{orient}(-\mathbf{u}_1, \mathbf{u}_2)$. To transform a vector $\mathbf{u}_A$ into $\mathbf{u}_B$ we could rotate the former around the axis $\mathbf{e} = \text{sign}(\mathbf{u}_A \bullet \mathbf{u}_B)\, (\mathbf{u}_A \times \mathbf{u}_B) / \| \mathbf{u}_A \times \mathbf{u}_B \|$ using Rodrigues' rotation formula [2], generating a geodesic in the space of orientations. The interpolation of tensors can thus be defined by linear interpolation of λ_{long} , λ_{trans} from tensors **A** and **B**, and the orientation **u** that results from rotating $\mathbf{u}_A$ around **e** an angle that is a fraction of θ.

The regularization of tensor fields in the Total Variation framework [4] can be posed as the minimization of a functional that sums over distances between the tensor at each voxel and its neighbors. This minimization is implemented as a gradient flow that respects the mathematical structure of the tensor field, without the need of projections or flipping orientations. This regularization has been used on (a) a synthetic field, and (b) a phantom made of polyamide fibers wound around a roller [3], similar to in vivo neuronal tissue, and (c) a brain scan from a healthy volunteer (not shown in this abstract).

Datasets were acquired with a SE-EPI diffusion sequence on a 1.5T clinical scanner (Intera, Philips). Parameters: b = 800 s/mm^2, 15 diffusion directions, TE = 58 ms , TR = 8.5 ms, 3 averages, voxel size = 1.75×1.75×2.5 mm^3. Diffusion tensors were estimated and diagonalized using homemade software developed in Python.

Results

Figure 1 shows the result of the regularization on a synthetic orientation field (SNR = 5), where three regions of clear directionality are reconstructed (there is fourth region dominated by noise). Figure 2 shows a colormap from the cylindrical phantom (color represents principal orientation of diffusion ellipsoids), before and after regularization. Some improvements can be appreciated at specific voxels (only voxels of high anisotropy were modified by the algorithm).

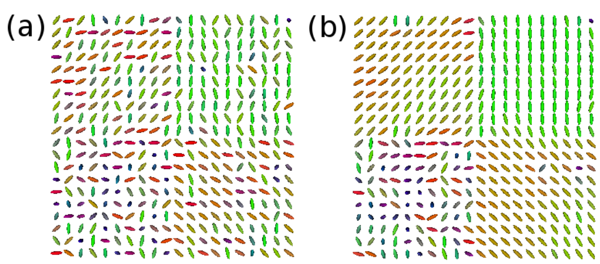

Fig. 1: (a) Noisy synthetic orientation field, (b) field regularized with Total Variation method.

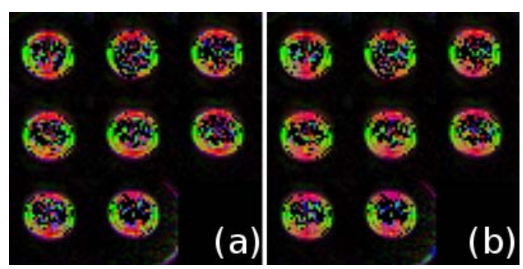

Fig. 2: (a) Noisy colormap of the diffusion phantom, (b) colormap of regularized tensor field.

Discussion

The present framework extends the notion of distance to the orientation of axially-symmetric tensors. This idea can be applied to their interpolation and regularization. For this last process, we were able to reduce noise of DTI datasets and increase the directional homogeneity in fiber bundles of a phantom. This effect may prove useful for tractography in regions of high SNR. The method can be extended to non-degenerate ellipsoids, taking into account the second and third eigenvectors.

References

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