A Simple but Robust Isotropic and Background Gradient Independent Diffusion Gradient Design

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Introduction

To accurately measure apparent diffusion coefficient (ADC), such as in stroke and functional diffusion imaging, background gradients from inhomogeneous magnetic fields or local susceptibility difference between tissues need to be considered in the diffusion gradients design. Various methods have been proposed to reduce the magnetic field interference [1-4]. Meanwhile, Wong, et al. [5] proposed isotropic diffusion weighting for measuring the trace of diffusion tensor, which provided imaging efficiency by reducing gradients directions. However, due to the computation complexity, there is no gradient design satisfying background field independent and orientation invariant requirements simultaneously. In this abstract we propose a theoretical framework for designing isotropic and background independent diffusion gradients.

Theory and Design

To design isotropic and background magnetic independent diffusion gradients, at least the following equations should be satisfied:

$$\int_0^t (k_i \cdot k_j)dt'' = 0 \quad (i \neq j, \text{ with i or j=1, 2, 3}) \qquad (1)$$

$$\int_0^t (k_i \cdot k_b)dt'' = G_b \int_0^t [\int_0^{t''} G_i(t')dt' \cdot t'']dt'' = 0 \qquad (2)$$

$$\int_0^t G_i(t')dt' = 0 \qquad (3)$$

Here k represents k-space trajectory from either diffusion gradients G_i or background gradients G_b. Eq. (1) requires the diffusion gradients in three directions are orthogonal to each other; eq. (2) restricts the cross-term between the applied diffusion gradients and any constant background magnetic fields to be zero; and eq. (3) requires the equilibrium of the applied gradients. To find the numerical solutions for this equation set, an exhaustive search algorithm is required. To avoid the computational demand, a simple alternative is proposed in this study.

For the isotropic diffusion gradient proposed by Wong, et al. [4], it has an extra property which was not described explicitly. If the background gradients in three directions x, y and z are the same [1, 2], it can be proven that such a setup is insensitive to cross-terms between the G_b and G_i. Our computer simulation shows that the ratio between the summation of the cross-terms and the total b factor from the applied gradients is $(b_x + b_y + b_z)/b_{xyz} = \text{-}2.92\% G_b / G$, where b_i (*i: x, y, z*) is the cross-term, b_{xyz} is the total b factor from the three-direction diffusion gradients, and G is the diffusion-gradient magnitude. The summation of the k-space trajectory product between the G_b and the G_i is shown in Figure 1, in which $b_x + b_y + b_z$ is proportional to the area of the plot according to eq. (2). Since the cross-term summation is only $2.92\% G_b / G$ of the applied b factor, it can be ignored in the diffusion weighting (assuming $G_b<G$). Thus these isotropic diffusion gradients are independent of the background gradients if they are the same along three directions.

When the background magnetic fields are different in three directions, which is more common realistically, a generalized diffusion gradient can be derived based on the property proved above. A simple setup is to cascade the isotropic diffusion gradients from three directions, G_x, G_y and G_z, into one waveform and apply to three directions respectively (Figure 2). The calculations above have already shown that the combination of three-direction gradients can remove the cross-terms. Therefore, there are no background-gradient contaminations in this setup even if they are not the same along the three directions. To keep the isotropic property, three-direction gradients need to be arranged with different order along each direction, e.g., gx=[G_x G_y G_z], gy=[G_y G_z G_x], gz=[G_z G_x G_y] (Figure 2). Thus along the transversal direction, the gradients have three segments and each segment satisfies eq. (1). With cross-terms canceled out and background gradient themselves having constant diffusion weighting, the proposed gradient sets can be used in any sequences and provide more accurate ADC measurements.

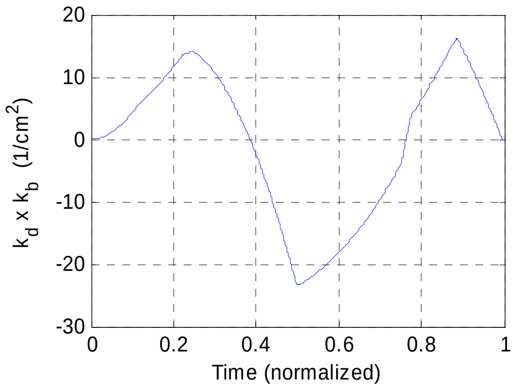

Figure 1. Three-direction summation of the k_i x k_b between the background gradients and the isotropic diffusion gradients. The b factor will be proportional to the area of the plot, which can be negligible compared to the applied b factor.

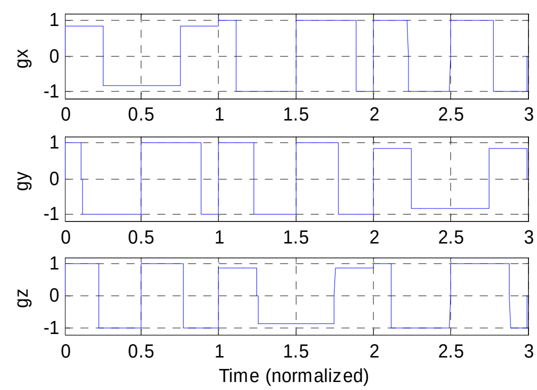

Figure 2. Isotropic diffusion gradients. They are from the cascade of three isotropic diffusion gradients with different permutations. In each direction they are independent of the background diffusion gradient. Along the transversal direction they are mutually orthogonal to each other.

Discussions

The proposed design can potentially provide the most accurate ADC measurements, although its diffusion weighting efficiency decreases (18.3% compared with a bipolar gradient) due to the oscillating gradient waveforms. It has been discussed that the background magnetic fields can contaminate the ADC changes during hemodynamic responses upon stimuli [3, 6]. This new diffusion gradient scheme could be very useful in functional ADC imaging to eliminate the background gradients effects on ADC changes. Additionally, this design has potential use in clinical imaging to reduce the susceptibility effects induced by metal implants.

References

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