Ellipsoidal Area Ratio (EAR): an Alternative Anisotropy Index for Diffusion Tensor Imaging

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Introduction Magnetic Resonance Diffusion Tensor Imaging (MR-DTI) provides far superior contrast within white matter of the brain compared with conventional Magnetic Resonance Imaging (MRI) modalities. DTI data, however, are mathematically complex, making the processing and analysis of DTI data much more complicated than that of scalar images. To make DTI data more computationally tractable, investigators have developed a family of so-called Diffusion Anisotropy Indices (DAIs), which are measurements of DTI data with reduced dimensionality. Fractional Anisotropy (FA) is the most popular of the DAIs. FA suffers from several shortcomings; however, including its sensitivity to noise and its absence of a direct and intuitive interpretation of a tensor's morphology (the FA's magnitude indicates only the relative elongation or sphericity of a tensor). Because a tensor's shape represents the diffusion probability at each location where it is measured, it is desirable to have a DAI that more directly reflects a tensor's distinct morphological features, and for that reason we have developed the Ellipsoidal Area Ratio (EAR). Monte Carlo simulations demonstrate that EAR is more robust to noise than is FA, particularly in white matter, and consequently EAR provides higher signal-to-noise ratios (SNRs) than does FA, while maintaining similar contrast-to-noise ratios (CNRs).

Table 1. Comparison between tissue types.

ROI	EAR	FA	Improvement
WM	6.96	4.66	49%
GM	2.81	2.71	4%
CSF	2.05	1.97	4%

Methods Computation of EAR involves an integral and trigonometry, which is computationally expensive. We can approximate the true surface area, however, using the *Knud Thomsen* approximation:

$$S = 4\pi \left[\frac{1}{3} \times (\lambda_1^p \lambda_2^p + \lambda_1^p \lambda_3^p + \lambda_2^p \lambda_3^p) \right]^{1/p}$$

which has the least relative error (±1.061% worst case) when $p \approx 1.6075$. To normalize the surface area, we divide S by the surface area of a sphere with radius $\max(\lambda_1, \lambda_2, \lambda_3)$. Suppose $\lambda_1 \geq \lambda_2 \geq \lambda_3$. EAR is defined as $1 - S/(4\pi\lambda_1\lambda_1)$:

$$EAR = 1.0 - \left[\frac{1}{3} \times \left(\frac{\lambda_2^p}{\lambda_1^p} + \frac{\lambda_3^p}{\lambda_1^p} + \frac{\lambda_2^p}{\lambda_1^p} \times \frac{\lambda_3^p}{\lambda_1^p} \right) \right]^{1/p}$$

Because diffusion tensors are used to study nerve fibers, DAIs are most commonly compared as a function of an anisotropy index defined for cylindrical symmetry,[1,2] where $\lambda_1 \neq \lambda_2$ but $\lambda_2 = \lambda_3$: $A = (\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2 + \lambda_3)$.

As a general comparison, we first calculated both EAR and FA head to head for $A \in (0,1)$ (corresponding to a tensor that ranges in shape from a sphere to a needle. To study the noise sensitivity of both EAR and FA, we preformed Monte Carlo simulations with 90,000 and 200,000 repetitions with differing levels of noise added to the diffusion weighted measurements,[3] and then calculated their mean, SNR, and CNR[4] values.

Results The simulations showed that EAR is a DAI of enhanced intensity than is FA in terms of cylindrical symmetry in white matter (where usually FA ≥0.2 or 0.25, Fig.1). Whereas the CNRs of EAR and FA were similar (Fig. 2), EAR was consistently more robust to noise than was FA. EAR had consistently higher SNR than did FA (Fig. 3). Our statistical study with 5 real subjects also confirmed that EAR offered higher SNR in white matter, grey matter and CSF (Table 1).

Conclusion EAR is relatively more immune to noise than is FA, and it therefore seems to be a good alternative measure of diffusion anisotropy. As EAR nonlinearly enhances the detection of anisotropy in white matter (Fig. 1), using EAR instead of FA to scale the principal direction of tensors could allow easier and more robust fiber tracking.

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References [1] Ulug, A.M., JMRI, 1999. **9**: p. 804-813. [2] Conturo, T.E, MRM 1996. **35**: p. 399-412. [3] Skare, S, MRI, 18(2000)659-669. [4] Kingsley, P.B. MRM, 2005, **53**(4): p. 911-918.

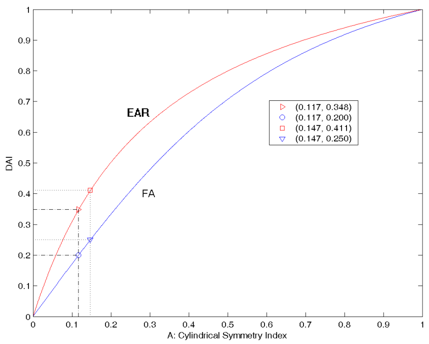

Figure 1: Head-to-head CNR comparison of EAR and FA in the range of A in [0,1]. The circle and triangle on the FA curve mark where FA = 0.2 and 0.25, respectively, for segmenting white matter in the brain.

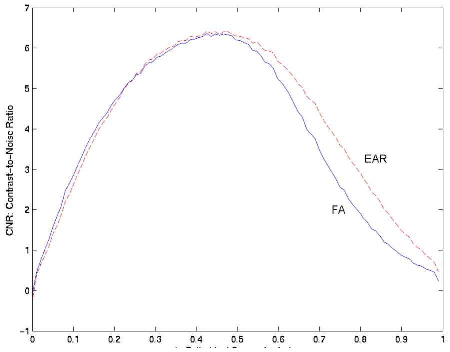

Figure 2. CNR of EAR and FA based on Monte Carlo simulation of 200,000 repetitions with noise superimposed at a level of 10% of the baseline diffusion weighted signal S_0 (corresponding to an SNR of 7.1 for S_0).

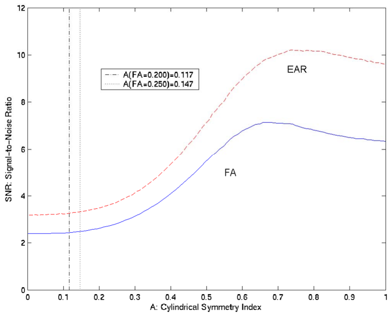

Figure 3. SNR of EAR and FA, based on the same Monte Carlo simulations in Fig. 2.