The Elliptical Cone of Uncertainty in Diffusion Tensor Imaging and Its Normalized Measures

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INTRODUCTION

The diffusion tensor is always measured in the presence of background noise. Recent experimental observations by Jeong et al.[1] and Lazar et al.[2,3] have shown that the cones of uncertainty (COU) [4,5,6] of the major eigenvectors of the diffusion tensors in the brain are mostly elliptical. Analytical approaches have also been developed to characterize the elliptical cone of uncertainty [7] through nonlinear least squares estimations [8]. Here, we present a new technique to construct the COU based on the inverse of the Gnomonic projection and two normalized geometric measures associated with the COU—the normalized areal and circumferential measures.

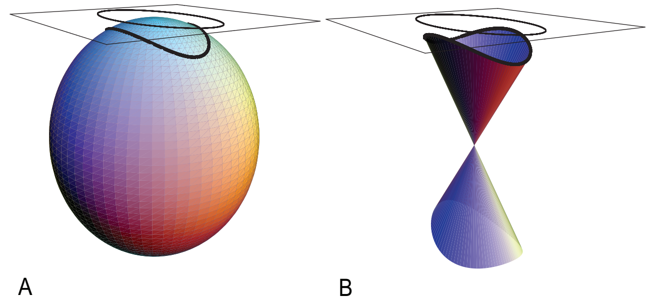

Fig. 1. (A) Inverse Gnomonic projection of an ellipse of the Cartesion plane onto the unit sphere is accomplished by normalizing the vector on the plane to unit length so that the normalized vector is on the surface of the unit sphere. (B) The proposed construction of COU is based on the inverse Gnomonic projection of an ellipse of the Cartesion plane onto the unit sphere.

METHODS

A simple closed or Jordan curve [9] on the unit sphere divides the unit sphere into two regions. If the simple closed curve is not the great circle then one region will be greater than the other. The normalized areal measure is the ratio of the area of the smaller region on the unit sphere, which is enclosed by a simple closed curve whose Gnomonic (or central) projection [10] on the Cartesian plane is an ellipse as depicted in Figure 1A, to the area of the hemisphere. The ratio of the circumference of the simple closed curve on the unit sphere to the circumference of the great circle of the unit sphere is the normalized circumferential measure. The inverse of the Gnomonic projection is used to construct the proposed COU, Figure 1B, that avoids overlapping cones in neighboring regions.

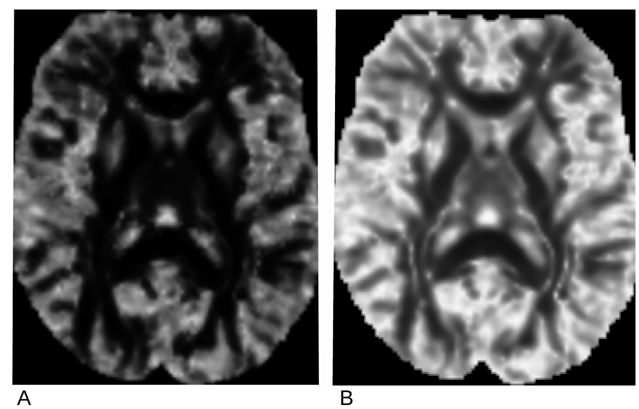

Fig. 2. (A) The map of the normalized areal measure and (B) the map of the normalized circumferential measure. The maps are generated from the 95% confidence COU at ant SNR level of 15.

RESULTS AND DISCUSSION

Simulated human brain diffusion tensor data were used to generate the normalized areal measure map, Fig 2A, and the normalized circumferential measure map, Fig 2B. The simulated data were generated from a single tensor model at an SNR level of 15. The normalized maps were computed from the 0.95 joint confidence region (or 95% confidence region), of the COU. Figure 3 shows the COUs of an axial slice of the simulated human brain using the proposed technique.

The normalized circumferential and areal measures are local parametric coherence measures for quantifying tract dispersion. The key advantage of the proposed measures for quantifying uncertainty of the major eigenvector of the diffusion tensor is that these measures are dimensionless and normalized to unity. Further, they have direct geometric interpretations.

Since the major eigenvector of the diffusion tensor is usually associated with the directional preference of the diffusing water molecules. The proposed measures, which are directly linked to the uncertainty in the major eigenvector of the diffusion tensor, may be important for probing the integrity of the white matter tracts in the brain and for assessing the quality and reliability of DTI tractography [11-15].

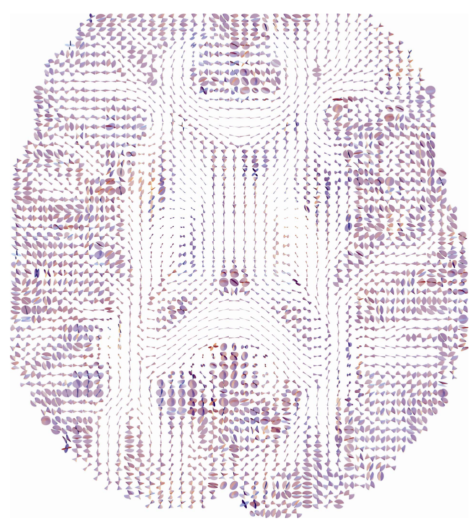

Fig. 3. An axial slice of the map of 95% confidence COU at an SNR level of 15.

REFERENCES

[1] Jeong et al. ISMRM p.1317 (2005). [2] Lazar et al. NeuroImage 2003; 20:1140-53. [3] Lazar et al. MRM 2005; 54: 860-7. [4] Basser PJ. Proc. ISMRM. p. 1740 (1997). [5] Basser et al. MRM 2000; 44: 41-50. [6] Jones DK. MRM 2003; 49 :7-12. [7] Koay et al. IEEE TMI 26; 8: 1017-34. [8] Koay et al. JMR 2006; 182: 115-125. [9] Apostol TM, *Mathematical Analysis*, 2nd ed., 1974. [10] Coxeter HSM, *Introduction to Geometry*, 2nd ed., 1969. [11] Mori et al. Annals of Neurology 1999; 45: 265-9. [12] Behrens et al. MRM 2003; 50: 1077-88. [13] Parker et al. JMRI, 2003; 18: 242-54. [15] Friman et al. IEEE TMI 2006 25; 8: 965-78.