Optimal number of excitations for a Rosette Spectroscopic Imaging Experiment

C. Schirda[1], and F. Boada[2]

[1]Buffalo Neuroimaging Analysis Center, State University of New York at Buffalo, Buffalo, NY, United States, [2]Radiology, University of Pittsburgh, Pittsburgh, PA, United States

Introduction: Rosette Trajectories [1-3] have been demonstrated in ^1H and ^{31}P spectroscopic imaging experiments and shown to achieve similar performance to the spirals. Their innate property of periodically sampling the center and edges of k-space and smoothly varying gradients makes for easy to design gradient waveforms. However, unlike spirals (or other non-Cartesian trajectories, e.g. PI), it is difficult to quantify analytically the number of excitations required based on the position of the sampled points, due to the way the k-space sampling density varies. We derive an analytical formula for the number of shots that provides for the highest sampling efficiency in a Rosette Spectroscopic Imaging (RSI) experiment.

Theory: Rosette Trajectories (Fig 1) consist of a radial oscillation with frequency f_1, which rotates at the same time with frequency f_2 in kx-ky space, and are mathematically described by: $\vec{k} = k_{max} \cdot \sin(\omega_1 t) \cdot e^{i \cdot \omega_2 t}$,

$$G_r(t) = k_{max} \cdot \omega_1 \cdot \cos(\omega_1 \cdot t) / \gamma\!\!\!/ , \quad G_\theta(t) = k_{max} \cdot \omega_2 \cdot \sin(\omega_1 \cdot t) / \gamma\!\!\!/$$

Here, k_{max} is the highest spatial frequency sampled and $G_{r,\theta}$ is the trajectory speed (gradient strength) in the radial direction and perpendicular on this direction. A number of shots N_{sh} uniformly distributed over 2π (angular separation $2\pi / N_{sh}$) is used to cover the kx-ky space. Based on the periodic sampling of k=0 (corresponding to the largest separation between samples along time axis), the RSI spectral bandwidth can be calculated (4). The multi-shot data acquired in between two successive k=0 crossings generate images corresponding to different echo times, from which the spectral information is recovered. An appropriate number of shots needs to be chosen such all spatial frequencies are properly sampled in each temporal slice (defined by successive k=0 crossings). For the Rosette trajectories, while the sampling density decreases as they move away from the center of k-space (k=0) - as is the case for spirals or projection imaging (PI), when they get closer to k=k_{max} , the density slightly increases (Fig 2). Because the sampling rate and speed along each trajectory are already designed to ensure proper sampling [3], we estimate the largest separation in kx-ky space between two adjacent trajectories (C_1 and C_2) and constrain it to be equal to 1/*fov*.

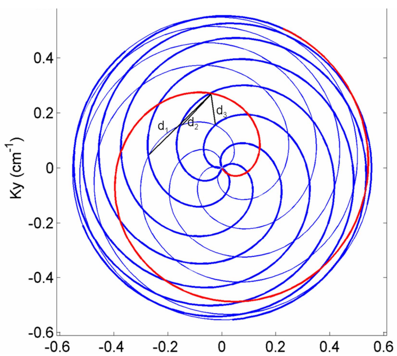

Fig1: *Rosette Trajectories in one temporal slice, shown in kx-ky space (* $\omega_2 / \omega_1 = 5.45$ *)*

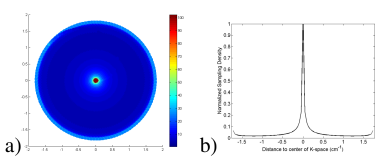

Figure 2: a) *kx-ky Rosette sampling density using Voronoi weights. b) Density Profile.*

Methods: In the kx-ky space of a temporal slice, each center-out trajectory segment intersects the out-in segments a number of (integer part of) $N_{cross} = N_{sh} \cdot \omega_2 / (2 \cdot \omega_1)$ times, for a total number of $N_{cross} \cdot N_{sh}$ crossings, arranged on N_{cross} concentric circles (with N_{sh} crossing on each circle, separated by $2\pi / N_{sh}$) with:

$$k_r^{(n)} = k_{max} \cdot \sin(\pi \cdot (1/2 - (n-1) \cdot (\omega_1 / \omega_2) / N_{sh})) \qquad n = 1, 2 \ldots N_{cross}$$

We define d(C_1,C_2)=max(min(d_1, d_2, d_3)), with d_1 the distance between two adjacent crossings on the same circle, d_2 the distance between two crossings on consecutive circles and corresponding to adjacent trajectories and d_3 the distance between crossings sitting on circles *n*,*n+2* on the same radial axis (Fig 1). It can be shown that:

$$d_1 \approx (2\pi / N_{sh}) \cdot k \; ; \quad d_2 \approx (\pi \cdot \omega_1 / \omega_2 \cdot N_{sh}) \cdot \sqrt{k_{max}^2 - (1 - (\omega_2 / \omega_1)^2) \cdot k^2} \; ; \quad d_3 \approx (2\pi \cdot \omega_1 / \omega_2 \cdot N_{sh}) \cdot \sqrt{k_{max}^2 - k^2}$$

Directly comparing d_1, d_2 and d_3, and setting d(C_1,C_2)=1/fov, yields:

$$N_{sh}(\omega_2 / \omega_1 \le 1) = \pi \cdot N_x / \sqrt{1 + 3 \cdot \omega_2^2 / \omega_1^2} \quad , \quad N_{sh}(\omega_2 / \omega_1 > 1) = \pi \cdot N_x / \sqrt{3 + \omega_2^2 / \omega_1^2}$$

Results: For N_x=64 and $\omega_2 / \omega_1 = .97$, the sampling efficiency [4] was measured using Voronoi volumes for: a) The minimum number of excitations for which Nyquist is still obeyed (as measured directly using a simulation program) N_{sh}^{min}=56 and up to the b) Estimated number N_{sh}^{est}=103. The actual weights (Voronoi), agree with theoretical ones $w(t) = G_r(t) \cdot dt \cdot G_\theta(t) \cdot dt \sim \sin(2 \cdot \omega_1 \cdot t)$, only as N_{sh} approaches N_{sh}^{est} , while η asymptotically increases to its theoretical value $\eta_{RSI} = .90$ (Fig 3).

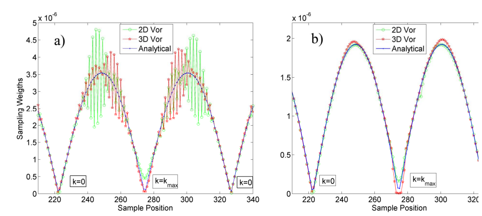

Figure 3: *Comparison of the theoretical pre-compensation weights (blue) to k-t space 3D Voronoi volumes (red) and 2D Vor (green) measured in kx-ky space in each temporal slice for a) Minimum number of excitations ->* $\eta = .88$ *and b) Estimated number of excitations ->* $\eta = \eta_{RSI} = .90$

Conclusions: We derived an analytical expression for the number of Rosette trajectories to be used in an RSI experiment that achieves the highest sampling efficiency. While this number is a slight overestimate (typically 20%), it provides for a reliable way to obtain a quick estimate for N_{sh}. In addition to maintaining the highest sampling efficiency, this number of shots represents a two-fold or greater reduction (depending on the amount of trajectory twist) compared to the number obtainable with a PI-like derivation (πN_x).

References: [1] Noll, IEEE Trans Med Imag, **16(4)**, '97 [2] Schirda et al. ISMRM Proc '03 [3] Schirda, Univ. of Pitt PhD Dissert'07. [4] Pipe et al, MRM **34(2)**, '95