Optimal Phased Array Combination for Spectroscopy

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Introduction Various methods to combine multiple spectra acquired by a phased array coil have been proposed (1–9). The basic approach is to form a weighted linear combination of the spectra or free induction decays (fid) using weights that ensure constructive addition of signals with maximum SNR. The model of the fid signal used in the present study is given by Eq 1, where A_j is the coil amplitude, ϕ_j is the coil phase, $s(t)$ is the time-varying MR signal and $\varepsilon_j(t)$ is noise.

$$S_j(t) = A_j \exp(i\phi_j)s(t) + \varepsilon_j(t) \qquad [1]$$

For simplicity, the noise is considered to be independently and identically distributed – there are standard ways of pre-processing the signals so that they conform to this model e.g. Refs (7,10). The fids comprise time-points t = 1,2,…,m and coils j = 1,2,…,n and the goal is to make a weighted linear sum of these fids (or spectra) to take maximum advantage of the SNR offered by the phased array. This is expressed in Eq 2 for complex weights w_j.

$$\sum w_j S_j(t) \qquad [2]$$

The various methods differ in how the weights are determined. Ref (7) proposed using the first time point of the fids to provide weights, i.e. $w_j = conj(S_j(t{=}1))$. The idea is to ensure the noise terms $\varepsilon_j(t{=}1)$ are "small" relative to the signal, which undergoes exponential decay. Others have suggested using a few points near the largest peak in the spectral-domain (6), or the area of the largest peak (3) or a linear combination of several peaks (4). Others have proposed using numerically modeled coil sensitivities or performing calibration scans (1,2,5) without water saturation to ensure a large signal (9). In this abstract, a method similar to Ref (8) is described and compared with Ref (7).

Theory The fid data $S_j(t)$ can be considered an $m \times n$ matrix (where m is no. time-points and n is no. coils). Denoting this as the data matrix **H** then, according to Eq 1, **H** comprises the sum of a signal term **S** and a noise term **N**.

$$\mathbf{H} = \mathbf{S} + \mathbf{N} \qquad [3]$$

The columns of **S** are just (complex) scalar multiples of the signal vector $s(t)$ and so **S** is a rank-1 matrix. The optimal rank-1 estimate is obtained from the noisy data by SVD: $\mathbf{H} = \mathbf{\Theta\Sigma\Omega}'$, followed by discarding all but the largest singular values. The principal column of $\mathbf{\Omega}$ contains optimal weights w_j – the identical weights are obtained from the principal eigenvector of **H'H**, which in Ref (10) is shown to follow from a formal maximization of the SNR.

Methods Spectra were acquired on a GE 3T TwinSpeed scanner using an 8-channel head coil and a spectroscopy head phantom. Water-suppressed STEAM was performed (TR 3000 ms, TE 10 ms, TM 13.7 ms, 5 kHz bandwidth, 2048 sampled points) with 2–64 NEX to vary the SNR. Raw fid signals were processed offline using MATLAB and reconstruction times were <1 second on a 3GHz personal computer.

Results & Discussion Visually it is difficult to assess noise levels, therefore two quantitative approaches were adopted. **(I)** Due to the linearity of the combination methods, the combined spectra can be described in terms of a signal term plus a noise term. The signal term is identical and so any differences are due to the noise. In particular, differences in the overall variance reflect differences in the noise variance – larger variance indicates higher noise. Table I gives the ratio of the variance in the SVD method above to the variance of the method of Ref (7) for different numbers of averages. In all cases, the variance is smaller in the SVD method (i.e. ratio < 1) indicating lower noise although as the SNR increases the methods become indistinguishable.

Table I

NEX	2	4	8	16	32	64
Ratio	0.88	0.81	0.88	0.85	0.94	0.97

(II) There is no reason to expect any difference in the weights between experiments other than due to noise fluctuations. The consistency of the weights determined in each experiment is an indication of how sensitive the combination method is to noise in the data – greater consistency indicates less sensitivity to noise. Table II lists the actual weights determined by the two methods in two different experiments. The weights vary considerably using the method of Ref (7) compared to those of the SVD method.

Conclusion The SVD method of combining coils yields higher SNR and is more consistent than the method of Ref (7).

Table II	**Weights determined by the method of Ref (7).**								**Weights determined by the SVD method above.**							
Coil j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$\lvert w_i \rvert$ **(expt 1)**	1.0	1.44	0.47	0.76	0.59	1.23	0.43	0.26	1.0	1.30	1.38	1.35	1.23	1.21	1.08	0.80
$\lvert w_i \rvert$ **(expt 2)**	1.0	0.99	1.06	0.47	0.97	1.34	0.79	0.45	1.0	1.31	1.26	1.25	1.25	1.23	1.03	0.82
$\angle w_i$ **(expt 1)**	0.0	-1.36	-1.44	-1.16	0.71	-0.70	-0.92	2.60	0.0	-0.80	-1.92	-2.39	0.57	-1.07	-0.80	-1.25
$\angle w_i$ **(expt 2)**	0.0	-1.25	-0.83	-1.08	1.16	-0.99	-0.68	-0.68	0.0	-0.77	-1.88	-2.37	0.63	-0.98	-0.70	-1.25

Refs **(1)** Hardy, Bottomley, Rohling, Roemer. *MRM* 1992;28:54 **(2)** Wald, Moyher, Day, Nelson, Vigneron. *MRM* 1995;34;440 **(3)** Maril, Lenkinski. *JMRI* 2005;21:317 **(4)** Wright, Wald. *NMR Biomed* 1997;10:394 **(5)** Schaffter, Bornert, Leussler, Carlsen, Leibfritz. *MRM* 1998;40:185 **(6)** Prock, Collins, Dzik-Jurasz, Leach. *Phys Med Biol* 2002;47:N39 **(7)** Brown. *MRM* 2004;52:1207 **(8)** Sandgren, Stoica, Frigo, Selen. *JMR* 2005;175:79 **(9)** Dong, Peterson. *MRI* 2007;25:1148 **(10)** Walsh, Gmitro, Marcellin. *MRM* 2000;43:682