Noise statistics, discretization artifacts, and SNR measurements at very low noise levels

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Introduction: Noise in MRI raw data is typically normally distributed in each receiver channel. The noise distribution in the final image depends on the reconstruction and channel-combination technique and is described e.g. by the Rayleigh or Rician distribution for single-channel data [1] and by the non-central chi-distribution in the case of a root-sum-of-squares (RSS) reconstruction [2]. However, these distributions only describe the probability density of real-valued (i.e. floating-point) intensity signals, while image data is typically discretized to integers before visualization or archiving in the DICOM format. Depending on the scaling factors used for the discretization and the signal-to-noise ratio (SNR), very low noise levels with substantial discretization artifacts can occur. The purpose of this study was to analyze the consequences of such discretization artifacts and to suggest an improved method for noise and SNR measurements in the presence of very low noise levels.

Methods: Complex signal data with Gaussian noise were simulated for different noise levels, σ, ranging from 0.02 to 1.50. From these, we calculated single-channel magnitude data and 16-channel RSS data. The data sets were discretized to positive integer values by truncation. We then calculated the mean value and standard deviation of the discretized data and estimated the originally applied noise level, σ_{est}, using the standard techniques for evaluation of background noise [3]. The results were compared to the original noise level by calculating the relative deviation σ_{est}/σ.

Our new technique for the determination of the original noise level, σ, is based on the evaluation of the relative frequency, f_s, of low pixel intensities with integer values, s, of 0, 1, 2, … in the background noise. We determined analytically (Box A) the frequency $F_s(\sigma)$ as a function of the original noise level, σ. $F_s(\sigma)$ can be used to estimate the noise level, σ, by fitting the relative frequencies f_s of low pixel intensities; the algorithm is described in more detail in Box B. This approach was evaluated in simulated image data with known original noise levels, σ = 0.30, 0.35, 0.40, …, 1.20, and in a T_1-weighted MR mammography image acquired with a 2-channel mammography coil and a routine protocol (Magnetom Symphony, Siemens Medical Sol., Erlangen, Germany).

Box A

Relative frequency of pixels with signal s (Rayleigh distribution) after truncation:

$$F_s(\sigma) = \int_s^{s+1} \frac{x}{\sigma^2} \exp\frac{-x^2}{2\sigma^2}\,dx = -\exp\frac{-x^2}{2\sigma^2}\bigg|_s^{s+1} = \exp\frac{-s^2}{2\sigma^2} - \exp\frac{-(s+1)^2}{2\sigma^2}$$

and for the non-central chi-distribution with n channels:

$$F_s(\sigma,n) = \int_s^{s+1} \frac{x}{(n-1)!\,\sigma^2}\left(\frac{x^2}{2\sigma^2}\right)^{n-1} \exp\left(\frac{-x^2}{2\sigma^2}\right) dx = -\exp\left(\frac{-x^2}{2\sigma^2}\right) \sum_{k=0}^{n-1} \frac{1}{k!}\left(\frac{x^2}{2\sigma^2}\right)^k \Bigg|_s^{s+1}$$

Box B

- Measure relative frequencies, f_s, of pixel intensities (s = 1, 2, 3, …, 20) in a background region

$$f_s = \frac{\#\text{pixels with intensity } s}{\#\text{pixels in region}}$$

- Fit the noise level σ by minimizing the weighted difference to the analytically determined frequencies (Box A):

$$\sigma = \min_{\sigma} \sum_{s=1}^{20} f_s \cdot \left(F_s(\sigma) - f_s\right)^2$$

Results: The deviations of the noise levels, σ_{est}/σ, determined from the mean value and standard deviation of background noise for single-channel and 16-channel data are shown in Fig. 1. The deviations are substantially larger than 10 % for small values of $\sigma<0.8$ and decrease for larger σ. The relative frequencies of low pixel intensities for single-channel and 16-channel RSS acquisitions are shown in Fig. 2a and 2b, respectively. The results demonstrate that the original noise level can be fitted (numerically or graphically) for $\sigma>0.30$ and $\sigma>0.15$ for single-channel and 16-channel data, respectively. Smaller noise levels cannot be determined since almost all pixel intensities are discretized to 0. The application of this approach in simulated image data reduced the mean deviation from 59.2 % (14.3 % with 16 channels) determined with the conventional mean-value-based calculation [3] to 0.4 % (0.05 %). The application in original MRI data is demonstrated in Fig. 3; the determined noise level is 0.65 in contrast to 0.40 and 0.86 derived from the mean value and standard deviation, respectively.

Conclusions: The suggested new technique significantly improves the accuracy of determined very low noise levels in MR images with discrete image intensities. Thus, less biased SNR determination becomes possible. The suggested approach can be applied not only to background noise but also to low-level difference images that are frequently used for SNR analysis in the presence of non-uniform image noise, e.g., in parallel-imaging applications [4]. In a retrospective analysis of archived MR image data at our site, we found several images with such low noise levels as in the shown MR mammography data set (Fig. 3).

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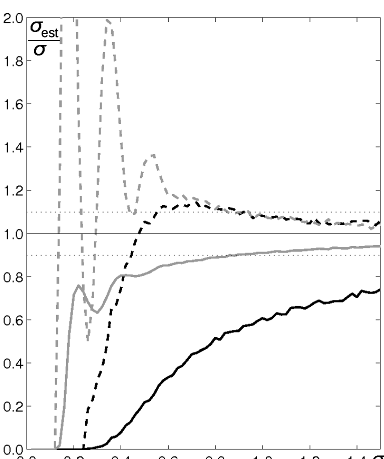

Fig. 1: Plot of relative deviations of the noise levels, σ_{est}/σ, estimated from the mean value (solid lines) and standard deviation (dashed lines) of background noise after discretization of single-channel (black) and RSS 16-channel (gray) data. The deviations are plotted as function of the real noise level σ.

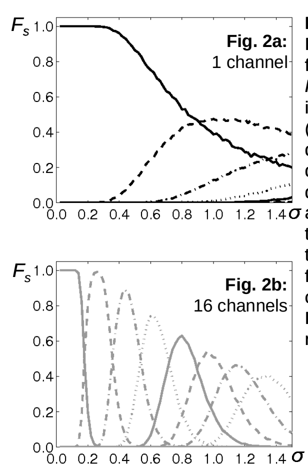

Fig. 2: Plots of relative frequencies, $F_s(\sigma)$, of low pixel intensities, s, (solid line: s=0, dashed line: s=1, dash-dotted: s=2, dotted: s=3, etc.) after discretization as function of the noise level σ for (a) single-channel and (b) RSS 16-channel reconstruction.

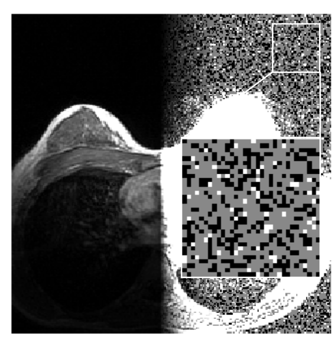

Fig. 3: Image example (MR mammography) demonstrating low noise levels in routine MRI data. Half of the image has been scaled up and a region has been magnified to visualize the background noise distribution: (black pixels: intensity s=0; gray: s=1, white s=2).