A Very Fast Reconstruction Algorithm for non-Cartesian Multi-coil Dynamic MRI

U. Sümbül¹, J. M. Santos¹, and J. M. Pauly¹

¹Electrical Engineering, Stanford University, Stanford, CA, United States

Introduction: Multi-coil imaging is one of the success stories of clinical MRI. Better image quality and/or shorter scan times have been demonstrated by various algorithms, SENSE[1] and GRAPPA[2] being the more popular ones. Non-Cartesian acquisitions, which generally have superior performance in the presence of flow and motion, present challenges to these techniques. One area where the robustness of non-Cartesian acquisitions can dramatically contribute is dynamic imaging since the object of interest(i.e, the heart) is likely to have complex temporal dynamics. However, dealing with a time series of images(instead of a single image) tremendously increases the difficulty of the already challenging problem. In this work, we present an algorithm that achieves very fast reconstruction rates on non-Cartesian multi-coil dynamic MRI. The algorithm is statistical in nature and uses both the spatial constraints(coil information) and temporal constraints(temporal behavior of image pixels) in the reconstruction. Such constraints have recently received attention in time-resolved applications. (See, for instance, kt-BLAST[3] and HYPER[4].) Consecutive frames from an in-vivo cardiac experiment with a 5-interleaf spiral readout trajectory are presented, demonstrating the merit of the algorithm with a 5x increase in the frame rate.

Theory: We use the dynamic system given in Fig. 1 to model the temporal variation of the image series. Here, S_n represents the true and noise-free image at time n, and G_n , F and Γ denote the time-dependent gridding operator, the Fourier transform operator and deapodization operator, respectively. C_k denotes the sensitivity matrix of the k^{th} coil, I_c is the c x c identity matrix, and \otimes denotes the Kronecker product. U_n represents the change in the image from time n-1 to n, and W_n is the observation noise with a covariance of $\Sigma = \sigma^2 I$. Both U_n and W_n are modeled as zero-mean random processes. X_n denotes the raw data obtained by one spiral interleaf at time *n*.

This formulation is in the standard linear dynamical system form, and hence we can directly apply the Kalman filter, a celebrated statistical signal processing algorithm[5]. By using the raw data of each single interleaf, which by itself corresponds to a severely aliased image, we expect the Kalman filter to provide an alias-free image for each interleaf. However, as suggested in [6], we will exclude the very center of the k-space, which is fully sampled at each interleaf, and reconstruct that part by gridding. This approach allows for approximations resulting in huge computational savings on the Kalman filtering equations because the problematic covariance matrices become approximately diagonal. Moreover, time consuming parts of the Kalman filter are rearranged so that they become scan-independent, yielding further computational savings.

Lastly, fully sampling the very center(~1% of the total k-space) provides auto-calibration ability, as frequently used in the MRI literature.

One interesting property of the presented dynamical system model is the simplicity of the first equation. It suggests that our ability in estimating the first and second moments of the process U_n determines the quality of the reconstruction. Perhaps more importantly, we do not make any assumptions on the motion of the object of interest. Therefore, the algorithm is well suited for imaging pathological cases such as arhythmia, and it should robustly work on any in-vivo object of interest. The diagonal entries of the covariance estimate of U_n obtained from a typical scan is shown in Fig. 2 in image format.

Methods: The RTHawk real-time system[7] is used with a fat-suppressed GRE pulse sequence, a 5-interleaf spiral readout and an 8-channel cardiac coil. Display FOV is 26 cm with a resolution of 2 mm. No ECG-gating or breath-holding was used. A small region around the center of the k-space is fully sampled by each interleaf, and the sampling density decreases slightly and linearly in the outer part for an SNR-efficient data acquisition. The statistical estimates required to initialize the Kalman filter are obtained from approximately ten-second-long scans. Alternatively, they may be obtained from the data itself, doing away with the extra initialization scan. The noise variance estimate(σ^2) is used as a parameter to trade image denoising for faster tracking.

Results: Fig. 3 shows 4 consecutive frames obtained from a healthy volunteer. The imaging slices were chosen to include valve leaflets. These frames capture the valve as it opens up and and the time between consecutive frames is 20.4 ms, corresponding to 49 frames per second. We note that the rapid motion of the valve is displayed with great detail. The valve moves a small amount between consecutive frames and it is crisply displayed in all of the frames. Reconstruction is very fast, the major components being 2 gridding and 2 Fourier operations per image.

$$\begin{array}{lll} S_n &=& S_{n-1} + U_n \\ \\ X_n &=& \left(\mathbf{G_n} \, \mathbf{F} \, \mathbf{\Gamma} \otimes \mathbf{I_c} \right) \left[\begin{array}{c} \mathbf{C_1} \\ \vdots \\ \mathbf{C_c} \end{array} \right] S_n + W_n \end{array}$$

Figure 1 – The dynamical system model



Figure 2 – Covariance estimate for U_n : Bright pixels indicate rapid temporal dynamics whereas dark pixels correspond to pixels that are relatively stable over time. Note that the contribution of very low harmonics is excluded in this motion map.



Figure 3 – 4 consecutive frames obtained using an 8-channel cardiac coil and a spiral readout trajectory. The reconstruction runs at 49 frames per second.

Conclusions: The reason parallel imaging works is that the coil sensitivity information imposes constraints on the reconstruction and effectively unaliases undersampled data. Temporal dynamics of pixels play a similar role in dynamic imaging. While these ideas have been around, combining them with non-Cartesian sampling strategies has been a challenge. We presented a statistical algorithm based on the Kalman filter that provides rapid reconstructions and tested it by imaging the fast moving cardiac valves. Initial results show very high temporal resolution, benefiting from the advantages of spiral trajectories. References: [1] Pruessmann KP, et al., Magn Reson Med, 42:952 – 962, 1999 [2] Griswold MA, et al., Magn Reson Med, 47:1202 - 1210, 2002 [3] Tsao J, et al., Magn Reson Med, 50:1031-1042, 2003

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